ICS 6N Computational Linear Algebra
Vectors and Matrices

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Scalar: numerical numbers

- For example: 1, 0.2, -100, 10.123
- The entire collection of real numbers is written as $\mathcal{R}$
- Strings are not real numbers
- Can be complex numbers C (we will see them later in the course)
- $x \in R \Leftrightarrow x$ is a real number

Two-dimensional vectors

- A two dimensional vector u has two components, written as

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

- Some examples: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -0.1 \\ 2.5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- The entire collection of two dimensional vectors is written as $\mathcal{R}^2$

Algebra defined on R^2

- Addition of two vectors

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$$

- Multiplication by a scalar

$$C \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} C \times x \\ C \times y \end{bmatrix}$$

Examples of vector algebra

- $\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$
- $3 \times \begin{bmatrix}1.2\\2.3\end{bmatrix} = \begin{bmatrix}3.6\\6.9\end{bmatrix}$
- $(-1) \times \begin{bmatrix}1\\2\end{bmatrix} + 2 \times \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}-1\\-2\end{bmatrix} + \begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$
- $\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}-2\\-3\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

More examples

- $\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}-1\\-2\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$
- $\begin{bmatrix}1\\2\end{bmatrix} /3 = \frac{1}{3} \times \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\\frac{2}{3}\end{bmatrix}$
- $3/ \begin{bmatrix}1\\2\end{bmatrix} =$ Not defined...
- $3 + \begin{bmatrix}2\\3\end{bmatrix} =$ Not defined...

Geometric description of R^2

Vectors can be represented by an arrow in a rectangular coordinate system.

Geometric interpretation of vector addition

Addition using the parallelogram rule

Geometric interpretation of scalar vector multiplication

Changes the length of the vector, and changes direction if it is multiplied by a negative number

Three dimension vector space R^3

- Examples of vectors in R^3

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -0.1 \\ 2 \\ 100 \end{bmatrix}$$

- Addition

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{bmatrix}$$

- Multiplication

$$C \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} C \times x \\ C \times y \\ C \times z \end{bmatrix}$$

Geometric Representation in R^3

We can still use multiplication and addition in analogous way as in R^2

- $$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^n$$

- Addition and Multiplication

$$a \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b \times \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} a \times x_1 + b \times y_1 \\ a \times x_2 + b \times y_2 \\ \vdots \\ a \times x_n + b \times y_n \end{bmatrix}$$

Matrix

- A matrix is a rectangular array of numbers, arranged in rows and columns.
- For example:
- $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & -2.5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ is called a 2×3 (read two by three) matrix.
- Each entry is referred to by two indexes (i, j), specifying the row and column of the entry in A
- a_{ij} : entry at i-th row and j-th column

Matrix

- In general, an $m \times n$ matrix has m rows and n columns.
- $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$
- a_{ij} : entry at i-th row and j-th column
- In Matlab, a_{ij} is written $A(i,j)$