ICS 6N Computational Linear Algebra

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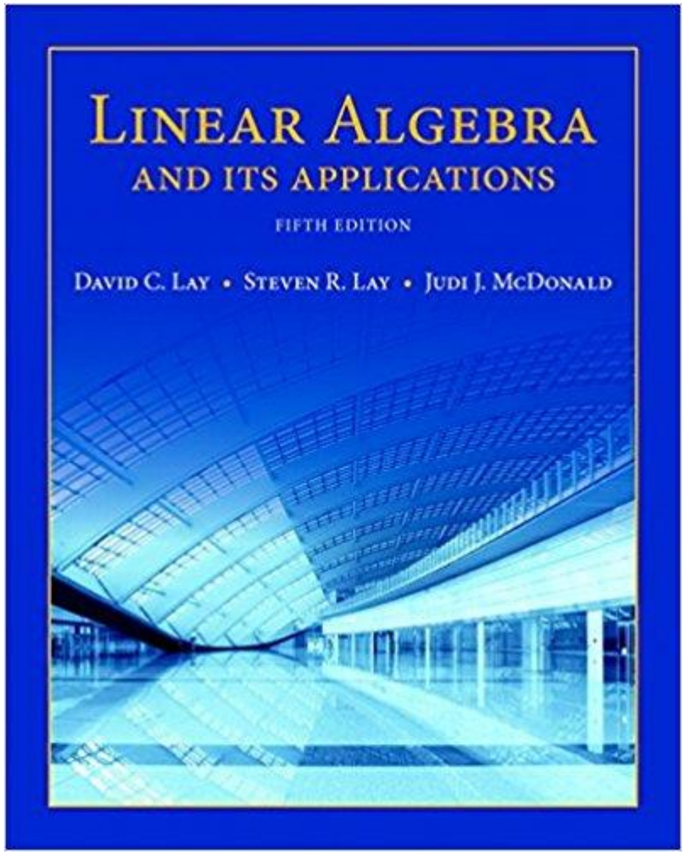
Teaching staff

- TA
  - Kanika Baijal kbaijal@uci.edu
- Readers
  - Liangjian Chen
  - Tiehang Duan
- Four discussion sessions (SBSG 241)
  - Discuss lecture material
  - Coding assignment

Textbook

Linear Algebra and its Applications, 5th. edition.
by David Lay.



However, you are only responsible for materials covered in lectures and discussion sessions.

Obtaining Assistance

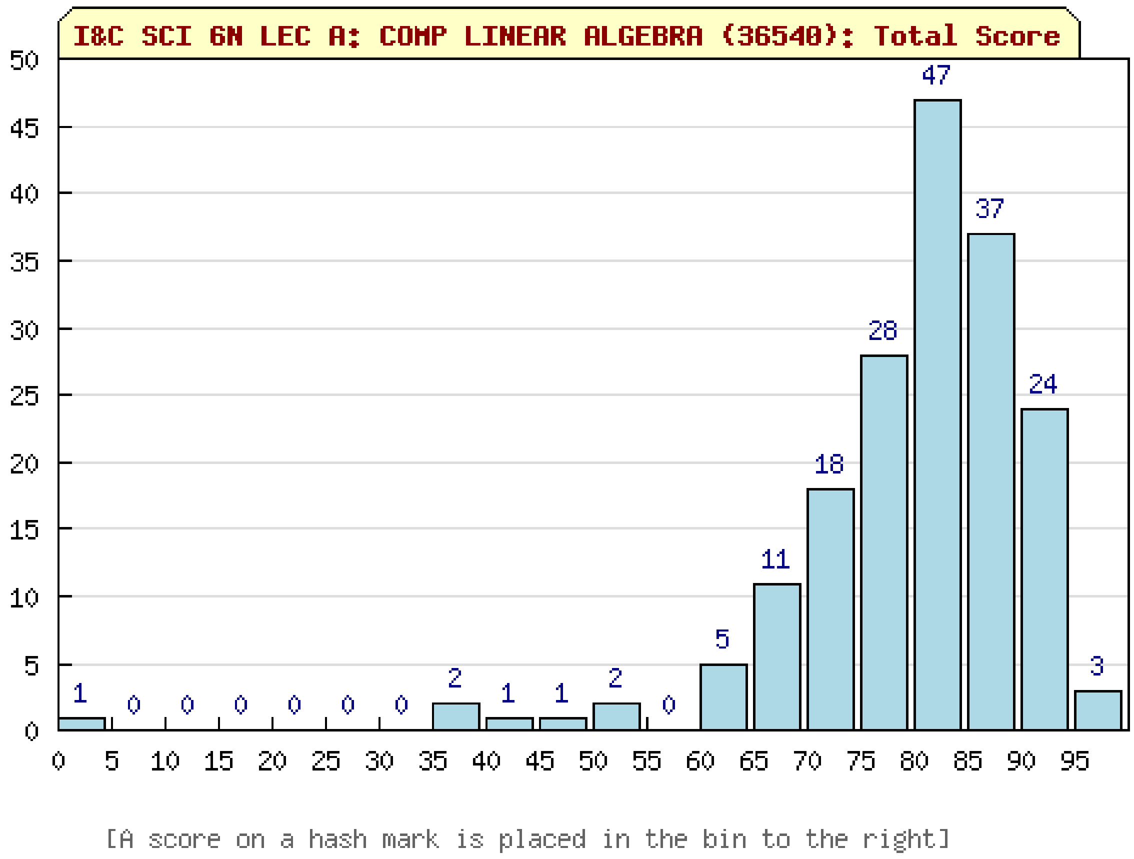
- Lecture and homework will be available from course website
- Use Piazza for class discussion
  - **https://piazza.com/uci/winter2018/ics6n/home**
  - The system is highly catered to getting you help fast and efficiently from classmates, the TA, and myself. Rather than emailing questions to the teaching staff, I encourage you to post your questions on Piazza.
- Email us for private questions

Grading criteria

- Homework (25%) (10)
- Lab assignments (15%) (4)
- Two quizzes (30%)
- Final (30%)

Please carefully read policies on course website regarding academic honesty!

Grade statistics from a previous year



Scale:
A ≥ 90 > A- ≥ 85
B+ ≥ 80 > B ≥ 75 > B- ≥ 70
C+ ≥ 65 > C ≥ 60

Topics

- Solving systems of linear equations
- Vector space, basis and dimension
- Least squares solutions
- Orthogonalization by Gram-Schmidt
- Properties of determinant
- Eigenvalues and eigenvectors
- Symmetric matrices and positive definite matrices
- Applications

Three components

- Notations: math vs. Matlab
- Algebra approaches
- Geometric approaches

Lecture 1

DATA TYPES, VECTORS, MATRICES

SCALARS

- Scalar is a real number
  - Examples: 1, 2.3, -0.2, 1005.6, 3.14159, -100
- If *x* is a real number, we usually say $x \in R$
- More specifically, ‘hello world’ is not a real number.
- In linear algebra, we primarily deal with continuous real numbers.
- We will get to know complex numbers at later part of the course.

VECTORS

Vectors in R^2

- Each vector consists of two real numbers
  - An example of a vector with two entries is $\mathrm{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

    where w_1 and w_2 are any real numbers.

- The set of all vectors with 2 entries is denoted by R^2 (read “r-two”).

VECTORS

- The $\mathbf{R}^2$ stands for the real numbers that appear as entries in the vector, and the exponent 2 indicates that each vector contains 2 entries.
- Two vectors in $\mathbf{R}^2$ are **equal** if and only if their corresponding entries are equal.
- Given two vectors **u** and **v** in $\mathbf{R}^2$, their **sum** is the vector **u+v** obtained by adding corresponding entries of **u** and **v**.
- Given a vector **u** and a real number *c*, the **scalar multiple** of **u** by *c* is the vector *c***u** obtained by multiplying each entry in **u** by *c*.

VECTOR EQUATIONS

- **Example 1:** Given $\mathrm{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\mathrm{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find

  4**u**, $(-3)\mathrm{v}$, and $4\mathrm{u} + (-3)\mathrm{v}$.

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  4**u**, $(-3)\mathrm{v}$, and $4\mathrm{u}+(-3)\mathrm{v}$.

Solution: $4\mathrm{u}=\begin{bmatrix} 4 \\ -8 \end{bmatrix}$, $(-3)\mathrm{v}=\begin{bmatrix} -6 \\ 15 \end{bmatrix}$ and

$$4\mathrm{u}+(-3)\mathrm{v}=\begin{bmatrix} 4 \\ -8 \end{bmatrix}+\begin{bmatrix} -6 \\ 15 \end{bmatrix}=\begin{bmatrix} -2 \\ 7 \end{bmatrix}$$

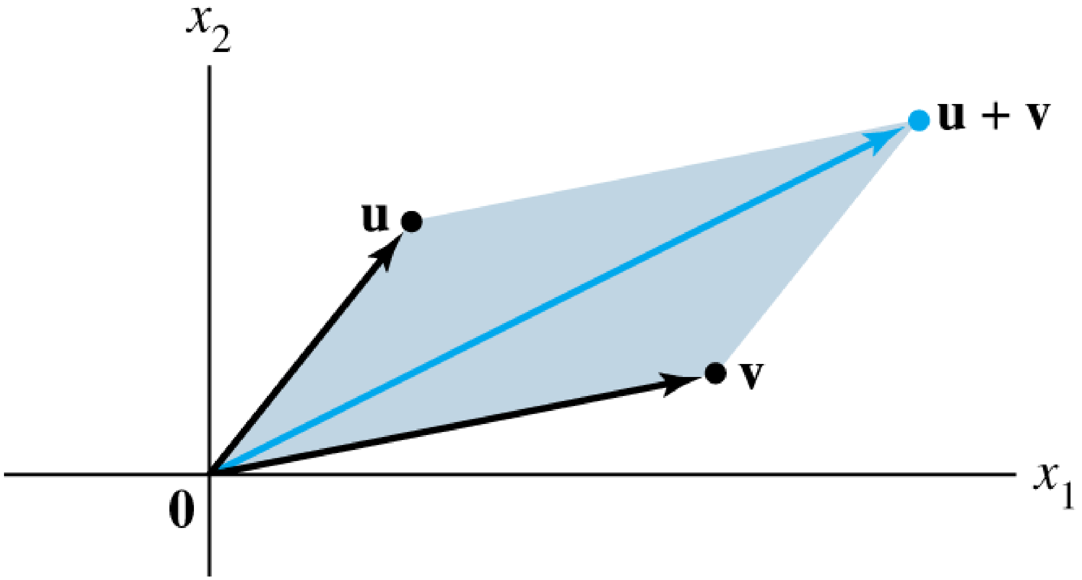
GEOMETRIC DESCRIPTIONS OF $\mathbf{R}^2$

- Consider a rectangular coordinate system in the plane. Because each point in the plane is determined by an ordered pair of numbers, *we can identify a geometric point (a, b) with the column vector* $\begin{bmatrix} a \\ b \end{bmatrix}$.

- So we may regard $\mathbf{R}^2$ as the set of all points in the plane.

PARALLELOGRAM RULE FOR ADDITION

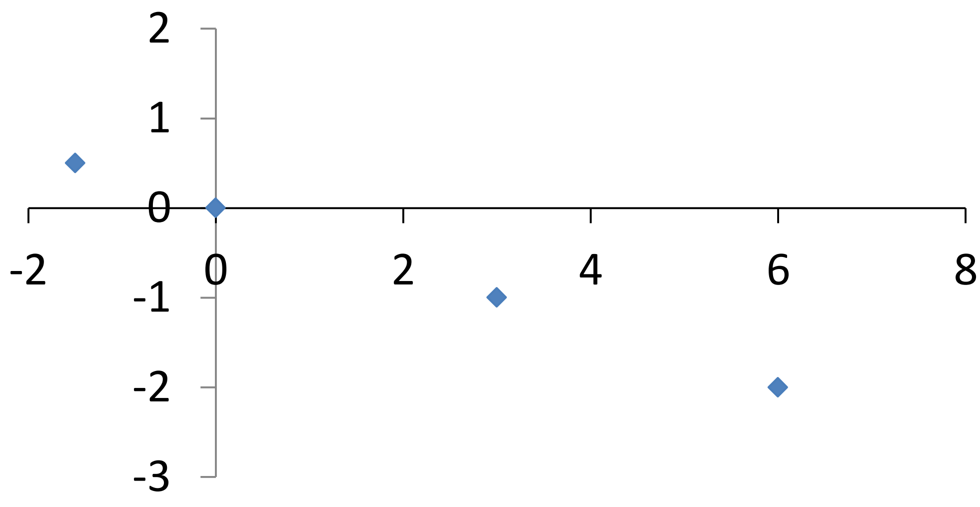
- If **u** and **v** in $\mathbf{R}^2$ are represented as points in the plane, then **u+v** corresponds to the fourth vertex of the parallelogram whose other vertices are **u**, **0**, and **v**. See the figure below.



Multiplication by a scalar

$u = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Display u, 2u, -0.5u



VECTORS IN $\mathbf{R}^3$ and $\mathbf{R}^n$

- Vectors in $\mathbf{R}^3$ are vectors with three entries.
- They are represented geometrically by points in a three-dimensional coordinate space, with arrows from the origin.
- If n is a positive integer, $\mathbf{R}^n$ (read "r-n") denotes the collection of all lists (or *ordered n-tuples*) of n real numbers, usually written as,

$$\mathrm{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$