

Name: _____ SUID: _____

Midterm Quiz

This is a 75 minute, closed notes, closed book midterm. Each question is worth 20 points. In questions 1–3, we will award 5 points per part for correct responses, and 2 points for parts left blank. For question 4, we’ll award some credit to partially correct responses. No justification is required for any of these questions, however.

By taking this quiz you’re agreeing to respect the honor code. Good luck!

1. *Convexity of some sets.* Determine if each set below is convex.

- (a) $\{(x, y) \in \mathbf{R}_{++}^2 \mid x/y \leq 1\}$
 convex not convex
- (b) $\{(x, y) \in \mathbf{R}_{++}^2 \mid x/y \geq 1\}$
 convex not convex
- (c) $\{(x, y) \in \mathbf{R}_+^2 \mid xy \leq 1\}$
 convex not convex
- (d) $\{(x, y) \in \mathbf{R}_+^2 \mid xy \geq 1\}$
 convex not convex

2. *Curvature of some functions.* Determine the curvature of the functions below. For affine functions (which are both convex and concave), select only the ‘affine’ box. If a function is neither convex nor concave, select ‘neither’.

- (a) $f(x) = \min\{2, x, \sqrt{x}\}$, with $\mathbf{dom} f = \mathbf{R}_+$
 convex concave affine neither
- (b) $f(x) = x^3$, with $\mathbf{dom} f = \mathbf{R}$
 convex concave affine neither
- (c) $f(x, y) = \sqrt{x \min\{y, 2\}}$, with $\mathbf{dom} f = \mathbf{R}_+^2$
 convex concave affine neither
- (d) $f(x, y) = (\sqrt{x} + \sqrt{y})^2$, with $\mathbf{dom} f = \mathbf{R}_+^2$
 convex concave affine neither

3. *Correlation matrices.* Determine if the following subsets of \mathbf{S}^n are convex.
- (a) the set of correlation matrices, $\mathcal{C}^n = \{C \in \mathbf{S}_+^n \mid C_{ii} = 1, i = 1, \dots, n\}$
 - convex not convex
 - (b) the set of nonnegative correlation matrices, $\{C \in \mathcal{C}^n \mid C_{ij} \geq 0, i, j = 1, \dots, n\}$
 - convex not convex
 - (c) the set of volume-constrained correlation matrices, $\{C \in \mathcal{C}^n \mid \det C \geq (1/2)^n\}$
 - convex not convex
 - (d) the set of highly correlated correlation matrices, $\{C \in \mathcal{C}^n \mid C_{ij} \geq 0.8, i, j = 1, \dots, n\}$
 - convex not convex
4. *DCP rules.* The function $f(x, y) = \sqrt{1 + x^4/y}$, with $\mathbf{dom} f = \mathbf{R} \times \mathbf{R}_{++}$, is convex. Express f using disciplined convex programming (DCP), limited to the following atoms,
- `inv_pos(u)`, which is $1/u$, with domain \mathbf{R}_{++}
 - `square(u)`, which is u^2 , with domain \mathbf{R}
 - `sqrt(u)`, which is \sqrt{u} , with domain \mathbf{R}_+
 - `geo_mean(u, v)`, which is \sqrt{uv} , with domain \mathbf{R}_+^2
 - `quad_over_lin(u, v)`, which is u^2/v , with domain $\mathbf{R} \times \mathbf{R}_{++}$
 - `norm2(u, v)`, which is $\sqrt{u^2 + v^2}$, with domain \mathbf{R}^2 .
- You may also use addition, subtraction, scalar multiplication, and any constant functions. Assume that DCP is sign-sensitive, *e.g.*, `square(u)` increasing in u when $u \geq 0$. Please only write down your composition. *No justification is required.*
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