EE 364a: Convex Optimization I R. Pathak
July 12, 2018

Name: ______________________ SUID: ______________________

Midterm Quiz

This is a 1 hour, closed notes, closed book midterm. Each problem is worth 12 points. In questions 1 and 3–5, you will be awarded 4 points per part for a correct response, and 2 points for parts left blank. No justification is required for these questions, however. For question 2, we'll award some credit to partially correct responses.

By taking this quiz you're agreeing to respect the honor code. Good luck!

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1. *True or false.* Write T or F if each statement is true or false. Suppose that $f, g : \mathbf{R}^n \to \mathbf{R}$ and $\phi : \mathbf{R} \to \mathbf{R}$ are given functions.

   (a) ____ If f, g are convex, then $h(x, y) = (f(x) + g(y))^2$ is convex.

   (b) ____ If f, ϕ are convex, differentiable, and $\phi' > 0$, then $\phi(f(x))$ is convex.

   (c) ____ If f, g are concave and positive, then $\sqrt{f(x)g(x)}$ is concave.

2. *DCP rules.* The function $f(x, y) = -1/(xy)$ with $\mathbf{dom}\, f = \mathbf{R}^2_{++}$ is concave. Briefly explain how to represent it, using disciplined convex programming (DCP), limited to the atoms $1/u$, $\sqrt{uv}$, $\sqrt{v}$, u^2, u^2/v, addition, subtraction, and scalar multiplication. Justify any statement about the curvature, monotonicity, or other properties of the functions you use. Assume these atoms take their usual domains (*e.g.*, $\sqrt{u}$ has domain $u \geq 0$), and that DCP is sign-sensitive (*e.g.*, u^2/v is increasing in u when $u \geq 0$).

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3. *Curvature of some functions.* Determine the curvature of the functions below.

(a) the product $f(u,v) = uv$, with $\mathbf{dom}\, f = \mathbf{R}^2$

☐ convex ☐ concave ☐ neither

(b) the function $f(x,u,v) = \log(v - x^T x/u)$, with $\mathbf{dom}\, f = \{(x,u,v) \mid uv > x^T x, u > 0\}$

☐ convex ☐ concave ☐ neither

(c) the 'exponential barrier' of polyhedral constraints

$$f(x) = \sum_{i=1}^{m} \exp\left(\frac{1}{b_i - a_i^T x}\right),$$

with $\mathbf{dom}\, f = \{x \mid a_i^T x < b_i, i = 1, \ldots, m\}$, and $a_i \in \mathbf{R}^n$, $b \in \mathbf{R}^m$

☐ convex ☐ concave ☐ neither

4. *Convexity of some sets.* Determine if each set is necessarily convex.

(a) $\{P \in \mathbf{R}^{n \times n} \mid x^T P x \geq 0 \text{ for all } x \succeq 0\}$

☐ convex ☐ not convex

(b) $\{(u,v) \in \mathbf{R}^2 \mid \cos(u+v) \geq \sqrt{2}/2,\ u^2 + v^2 \leq \pi^2/4\}$ (*Hint:* $\cos(\pi/4) = \sqrt{2}/2$)

☐ convex ☐ not convex

(c) $\{x \in \mathbf{R}^n \mid x^T A^{-1} x \geq 0\}$, where $A \prec 0$.

☐ convex ☐ not convex

5. *DCP compliance.* Determine if each expression below is (sign-sensitive) DCP compliant, and check the applicable box.

(a) `sqrt(1 + 4 * square(x) + 16 * square(y))`

☐ DCP convex ☐ DCP concave ☐ not compliant

(b) `min(x, log(y)) - max(y, z)`

☐ DCP convex ☐ DCP concave ☐ not compliant

(c) `log(exp(2 * x + 3) + exp(4 * y + 5))`

☐ DCP convex ☐ DCP concave ☐ not compliant