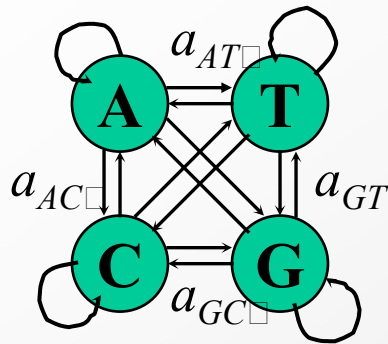


## **Example 1: Finding CpG islands**

# What are CpG islands?

- Regions of regulatory importance in promoters of many genes
  - Defined by their methylation state (epigenetic information)
- Methylation process in the human genome:
  - Very high chance of methyl-C mutating to T in CpG
    - ➔ CpG dinucleotides are much rarer
  - BUT it is suppressed around the promoters of many genes
    - ➔ CpG dinucleotides are much more frequent than elsewhere
      - Such regions are called **CpG islands**
      - A few hundred to a few thousand bases long
- Problems:
  - Given a short sequence, does it come from a CpG island or not?
  - How to find the CpG islands in a long sequence

# Training Markov Chains for CpG islands



- Training Set:
  - set of DNA sequences w/ known CpG islands
- Derive two Markov chain models:
  - **‘+’ model**: from the CpG islands
  - **‘-’ model**: from the remainder of sequence
- Transition probabilities for each model:

Probability of C following A

+	A	C	G	T
A	.180	.274	.426	.120
C	.171	.368	.274	.188
G	.161	.339	.375	.125
T	.079	.355	.384	.182

$$a_{st}^+ = \frac{c_{st}^+}{\sum_{t'} c_{st'}^+}$$

$c_{st}^+$  is the number of times letter  $t$  followed letter  $s$  inside the CpG islands

$$a_{st}^- = \frac{c_{st}^-}{\sum_{t'} c_{st'}^-}$$

$c_{st}^-$  is the number of times letter  $t$  followed letter  $s$  outside the CpG islands

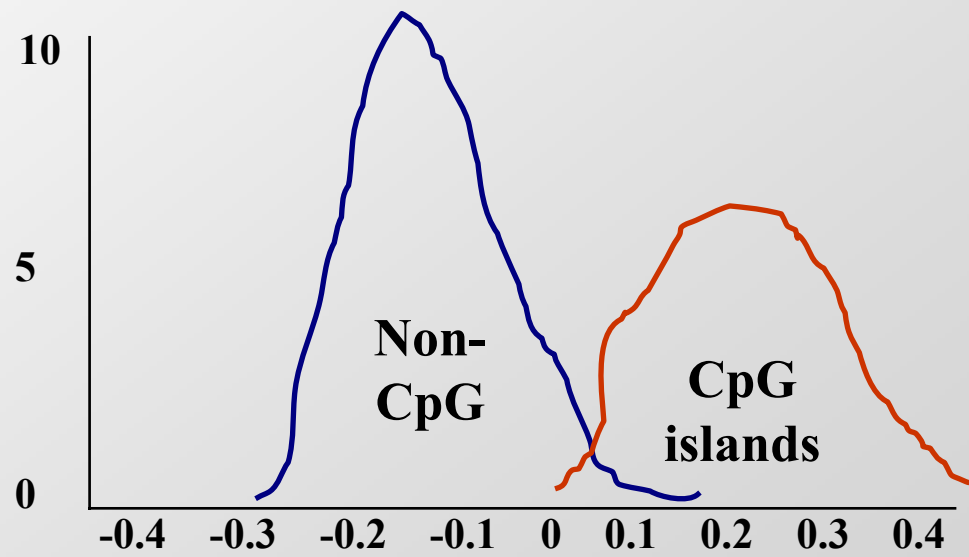
# Using Markov Models for CpG classification

Q1: Given a short sequence  $x$ , does it come from CpG island (Yes-No question)

- To use these models for discrimination, calculate the log-odds ratio:

$$S(x) \equiv \log \frac{P(x|\text{model } +)}{P(x|\text{model } -)} = \sum_{i=1}^L \log \frac{a_{x_{i-1}x_i}^+}{a_{x_{i-1}x_i}^-}$$

Histogram of log odds scores



# Using Markov Models for CpG classification

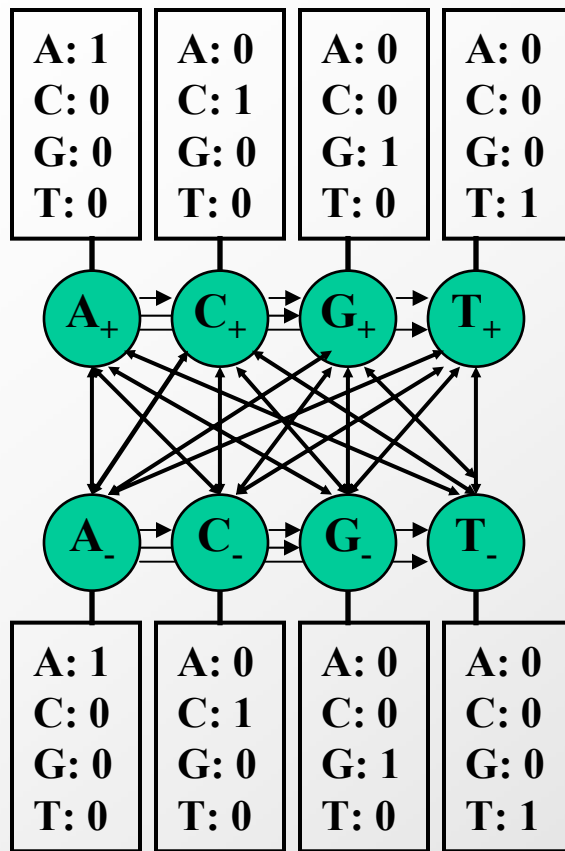
Q2: Given a long sequence  $x$ , how do we find CpG islands in it

(**Where** question)

- Calculate the log-odds score for a window of, say, 100 nucleotides around every nucleotide, plot it, and predict CpG islands as ones w/ positive values
- Drawbacks: Window size

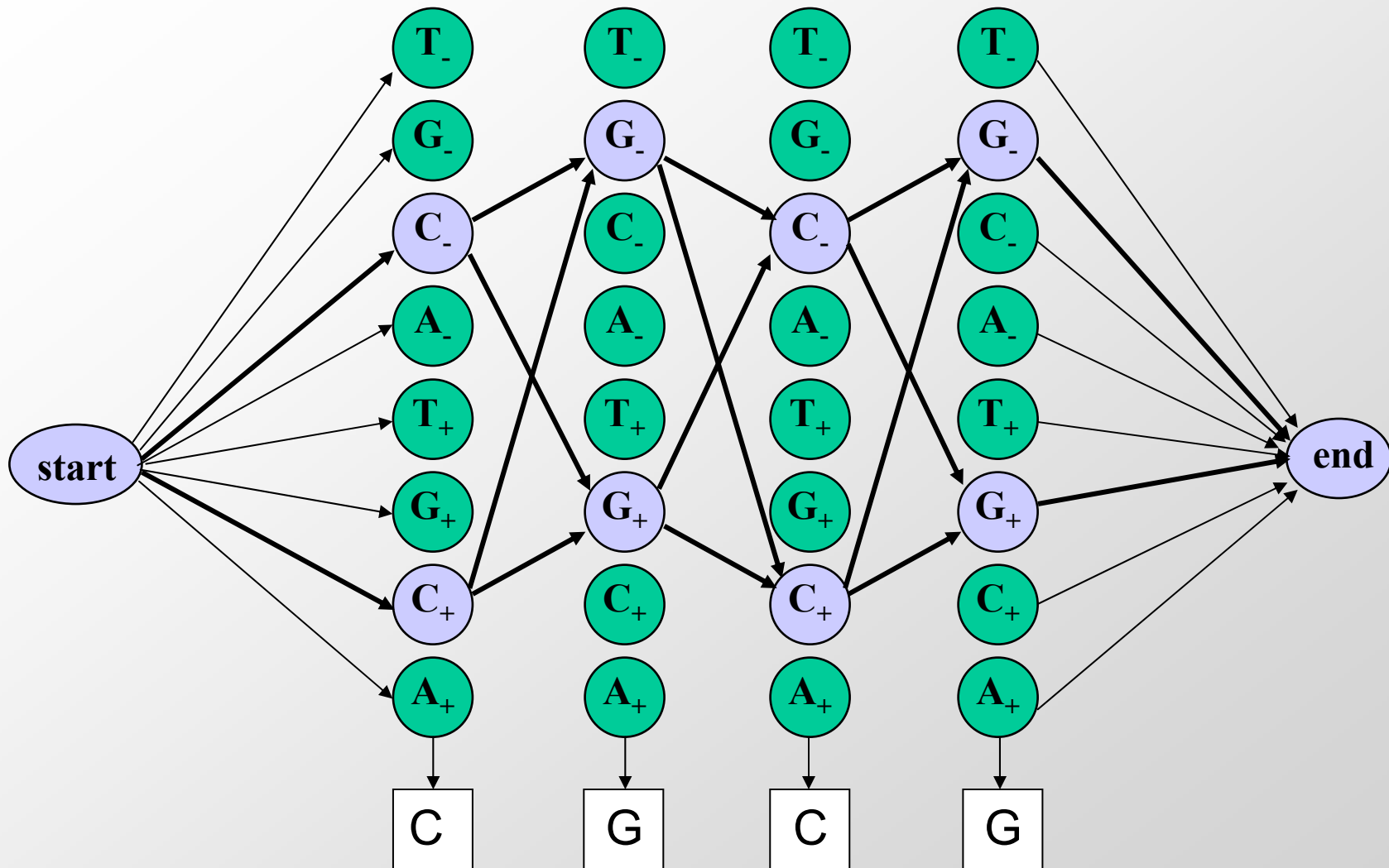
Use a hidden state: CpG (+) or non-CpG (-)

# HMM for CpG islands



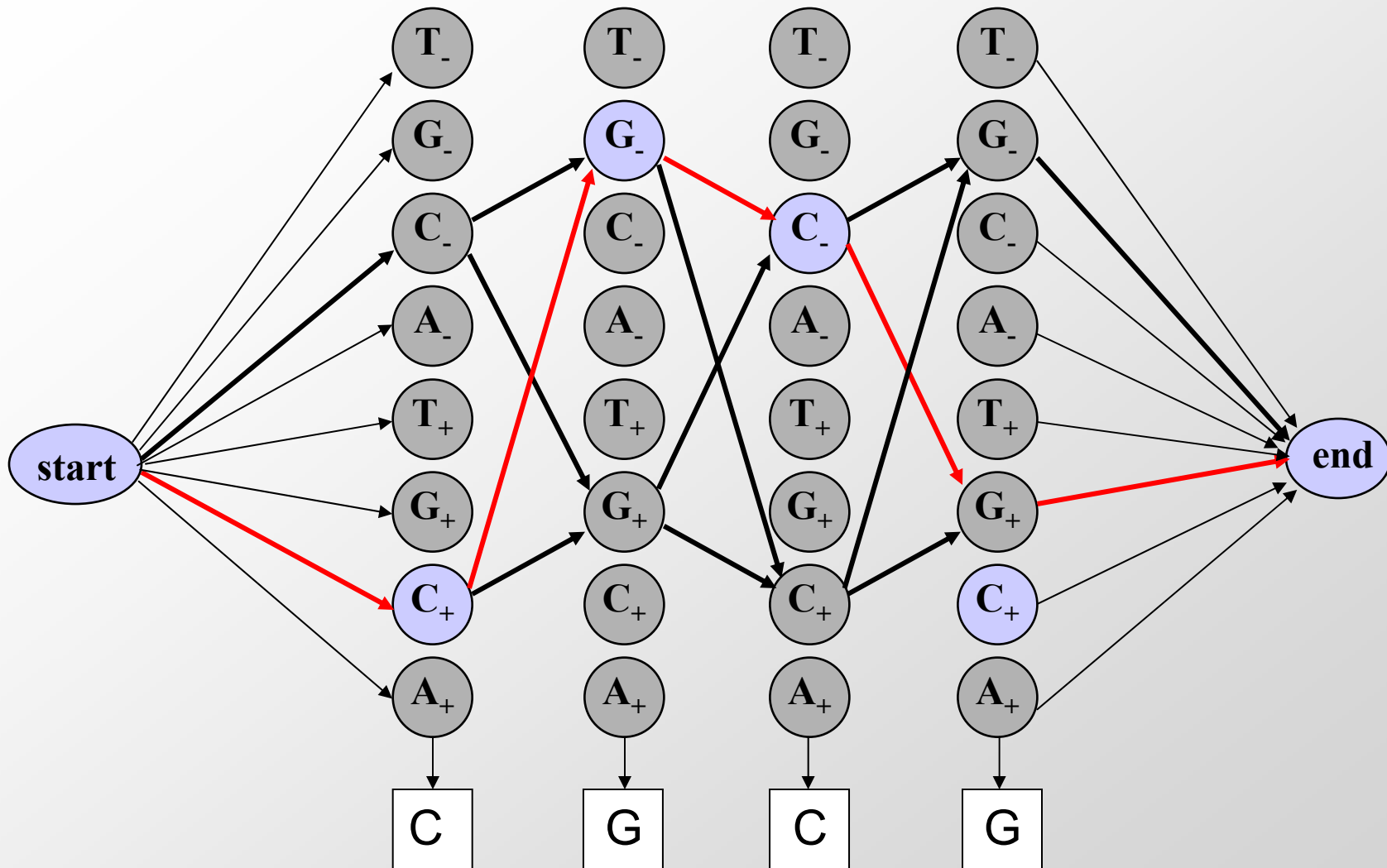
- Build a single model that combines both Markov chains:
  - ‘+’ states:  $A_+$ ,  $C_+$ ,  $G_+$ ,  $T_+$ 
    - Emit symbols: A, C, G, T in CpG islands
  - ‘-’ states:  $A_-$ ,  $C_-$ ,  $G_-$ ,  $T_-$ 
    - Emit symbols: A, C, G, T in non-islands
- Emission probabilities distinct for the ‘+’ and the ‘-’ states
  - Infer most likely set of states, giving rise to observed emissions
  - ➔ ‘Paint’ the sequence with + and - states

## Finding most likely state path



- Given the observed emissions, what was the path?

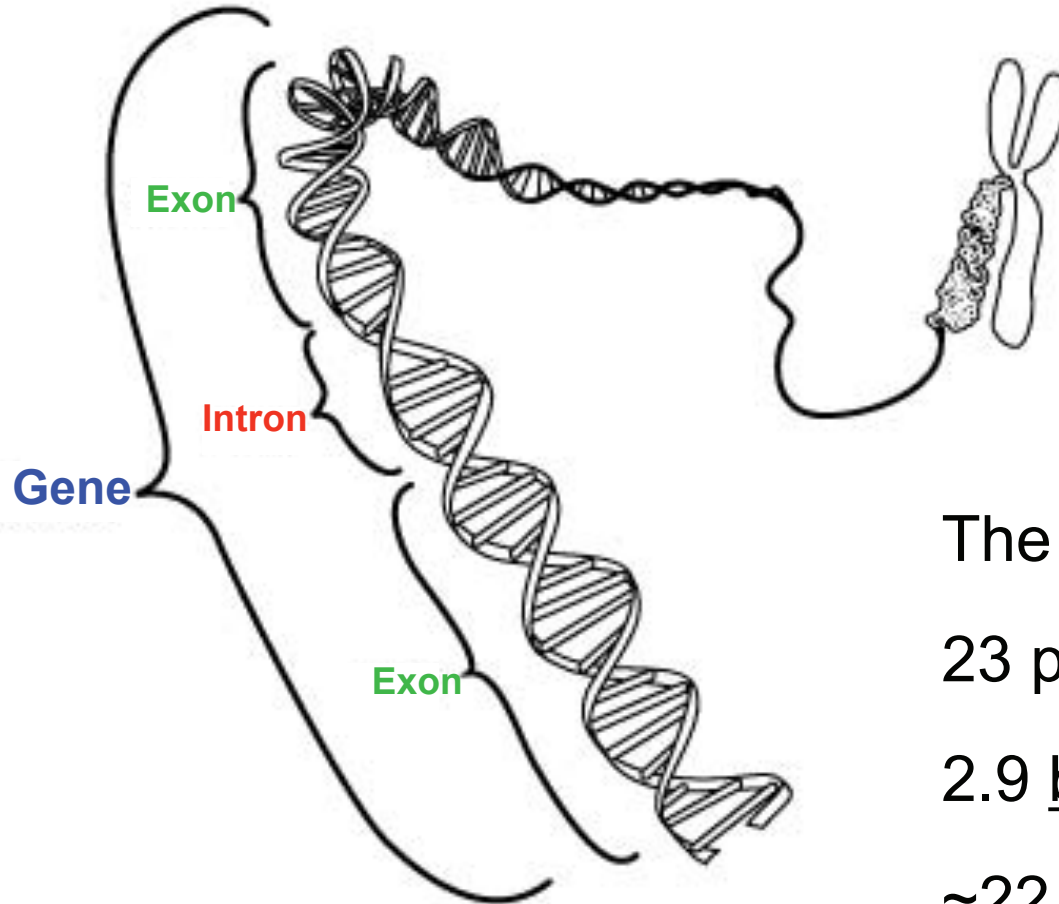
# Probability of given path $p$ & observations $x$



- Known observations: CGCG
- Known sequence path:  $C_+$ ,  $G_-$ ,  $C_-$ ,  $G_+$



# Exons, Introns, and Genes



The human genome:

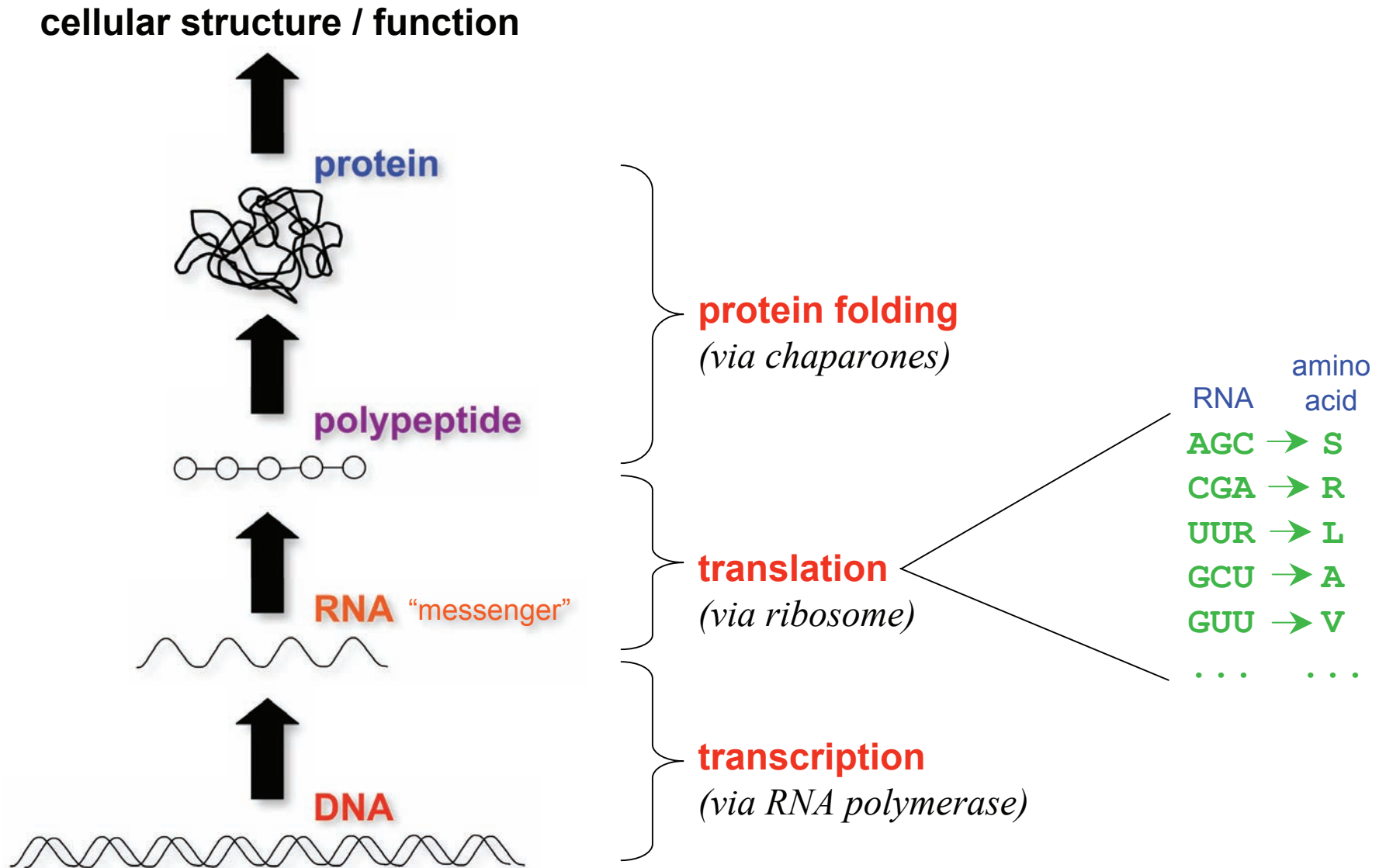
23 pairs of chromosomes

2.9 billion A's, T's, C's, G's

~22,000 genes (?)

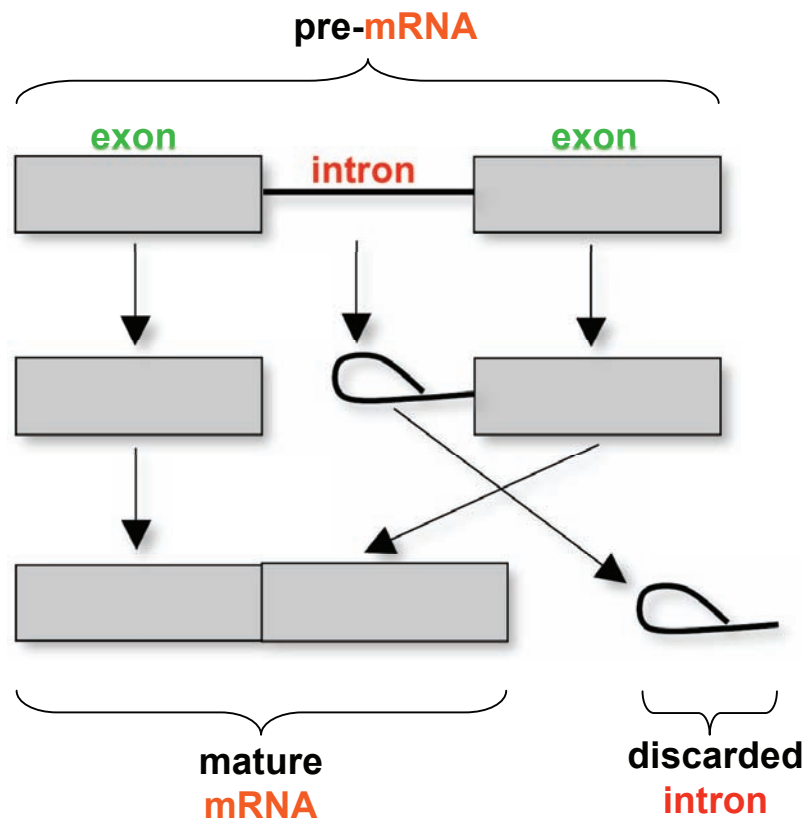
~1.4% of genome is coding

# The Central Dogma



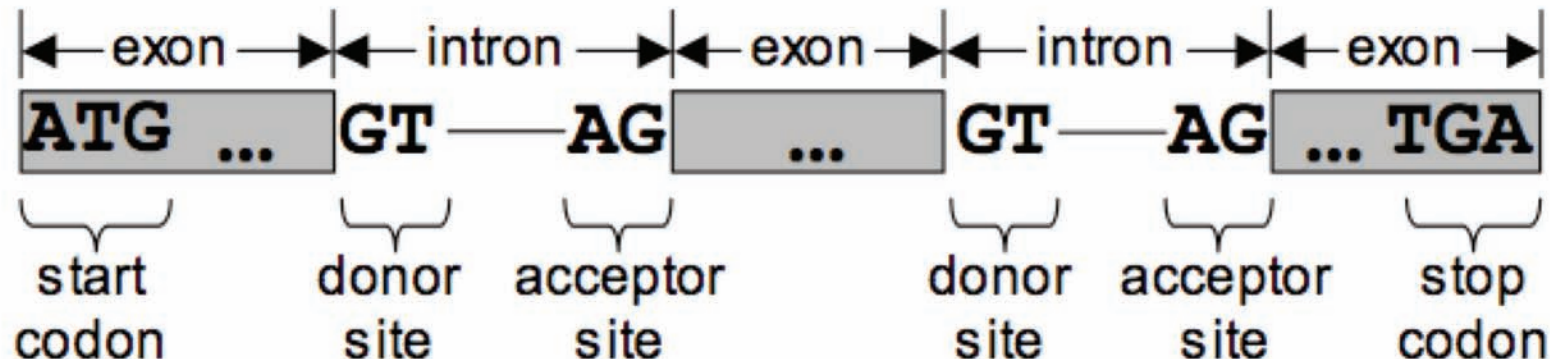
# Splicing of Eukaryotic mRNA's

After transcription by the *polymerase*, eukaryotic pre-mRNA's are subject to splicing by the *spliceosome*, which removes introns:



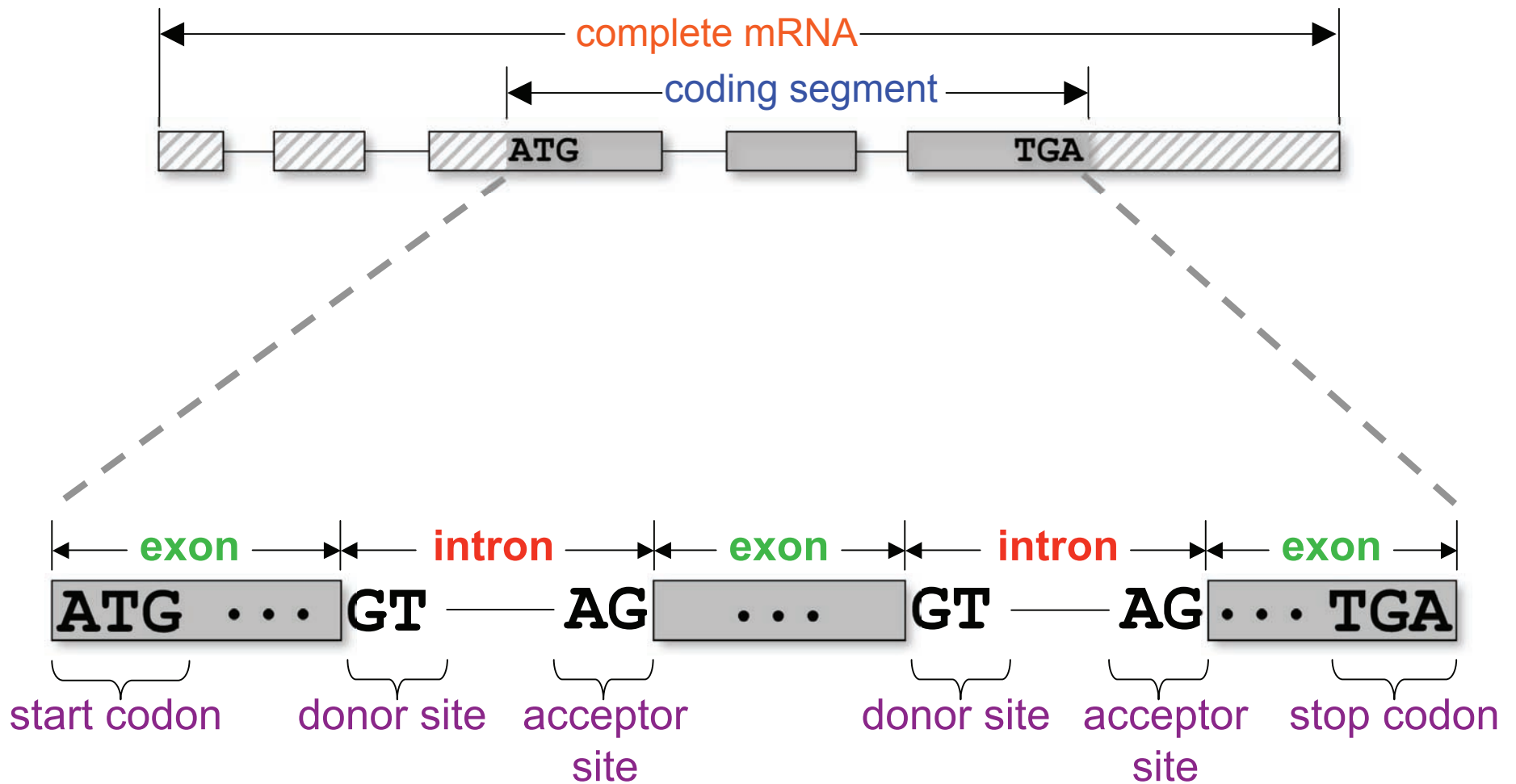
# Signals Delimit Gene Features

*Coding segments* (*CDS*'s) of genes are delimited by four types of signals: *start codons* (ATG in eukaryotes), *stop codons* (usually TAG, TGA, or TAA), *donor sites* (usually GT), and *acceptor sites* (AG):



For initial and final exons, only the coding portion of the exon is generally considered in most of the gene-finding literature; thus, we redefine the word “**exon**” to include only the coding portions of exons, for convenience.

# Eukaryotic Gene Syntax

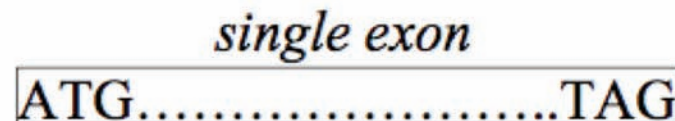
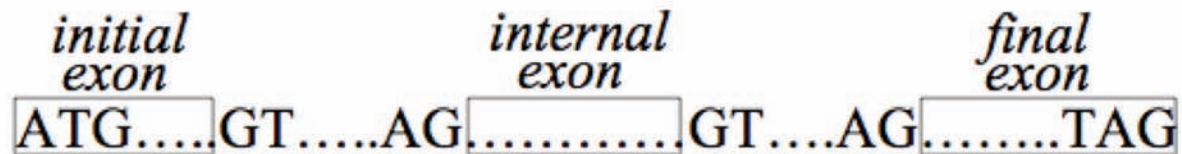


Regions of the gene outside of the CDS are called **UTR**'s (*untranslated regions*), and are mostly ignored by gene finders, though they are important for regulatory functions.

# Types of Exons

Three types of exons are defined, for convenience:

- *initial exons* extend from a start codon to the first donor site;
- *internal exons* extend from one acceptor site to the next donor site;
- *final exons* extend from the last acceptor site to the stop codon;
- *single exons* (which occur only in *intronless genes*) extend from the start codon to the stop codon:





# The Stochastic Nature of Signal Motifs

(start codons)

A T G



(stop codons)

T G A  
T A A  
T A G



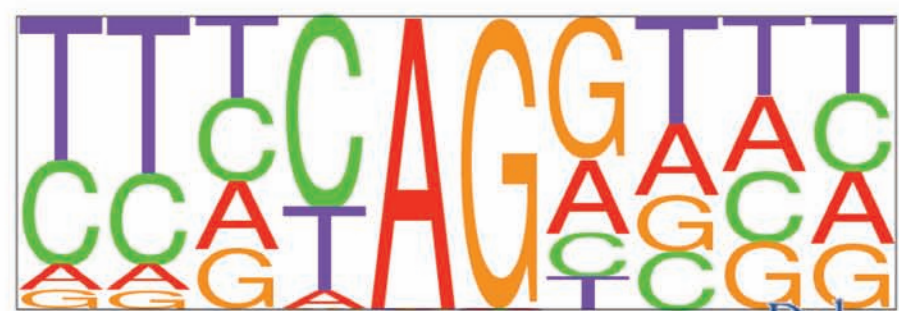
(donor splice sites)

G T



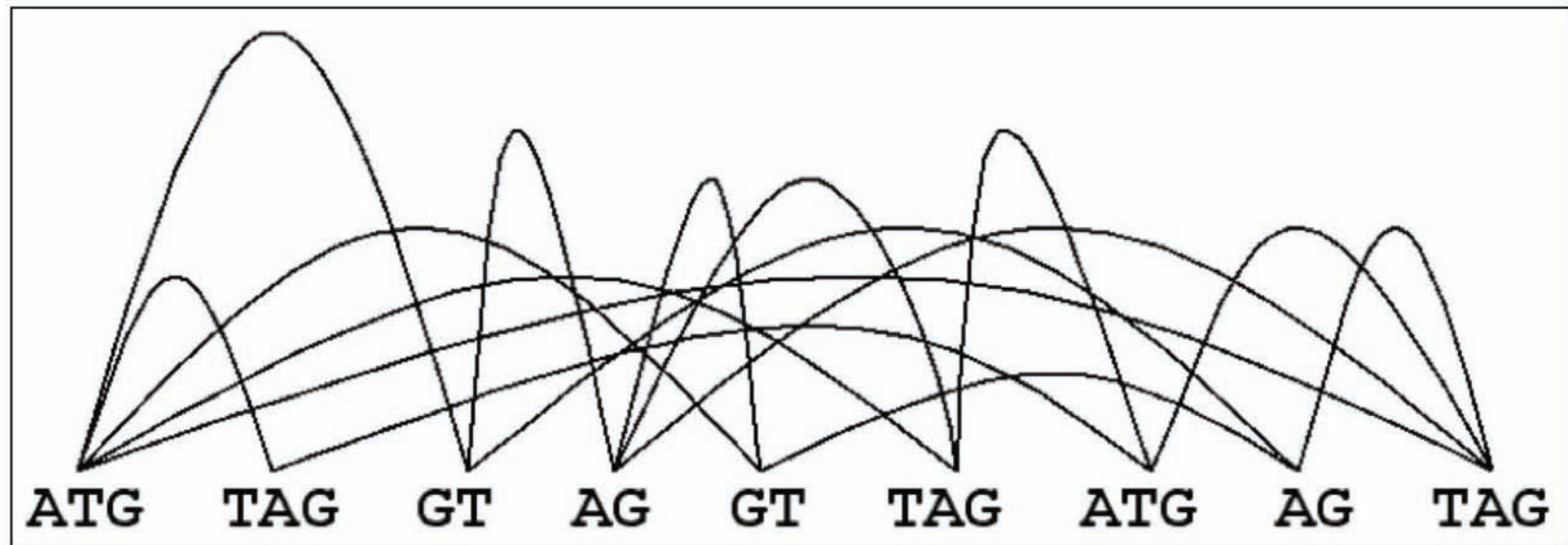
(acceptor splice sites)

A G



# Representing Gene Syntax with ORF Graphs

After identifying the most promising (i.e., highest-scoring) signals in an input sequence, we can apply the gene syntax rules to connect these into an *ORF graph*:

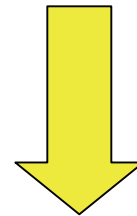


An ORF graph represents all possible *gene parses* (and their scores) for a given set of putative signals. A *path* through the graph represents a single gene parse.

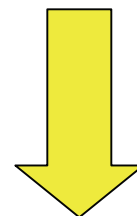
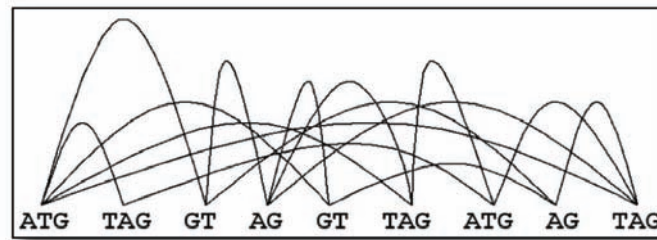


# Conceptual Gene-finding Framework

```
TATTCCGATCGATCGATCTCTCTAGCGTCTACG  
CTATCATCGCTCTCTATTATCGCGCGATCGTCC  
ATCGCGGAGAGTATGCTACGTCGATCGAATTG
```



identify most promising signals, score signals and content regions between them; induce an ORF graph on the signals



find highest-scoring path through ORF graph; interpret path as a gene parse = gene structure



# Recall: “Pure” HMMs

An HMM is a *stochastic machine*  $M=(Q, \alpha, P_t, P_e)$  consisting of the following:

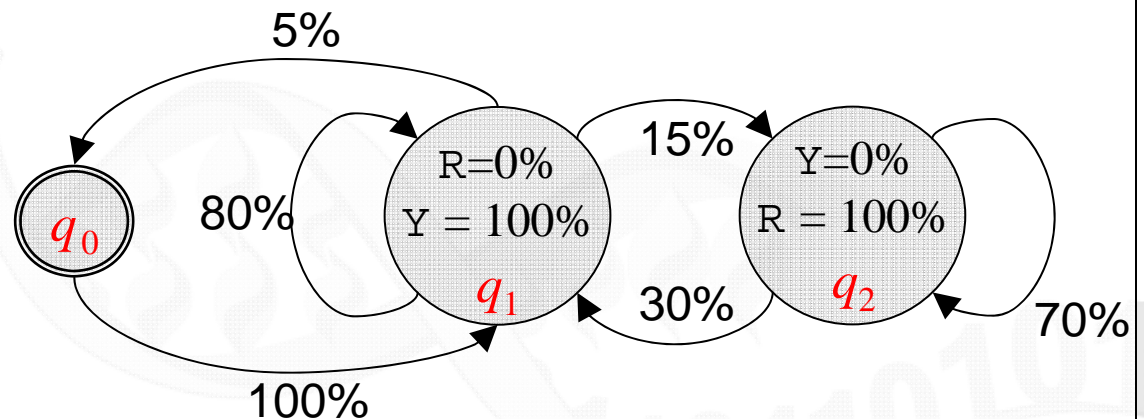
- a finite set of states,  $Q=\{q_0, q_1, \dots, q_m\}$
- a finite alphabet  $\alpha =\{s_0, s_1, \dots, s_n\}$
- a transition distribution  $P_t: Q \times Q \rightarrow [0,1]$  i.e.,  $P_t(q_j | q_i)$
- an emission distribution  $P_e: Q \times \alpha \rightarrow [0,1]$  i.e.,  $P_e(s_j | q_i)$

## An Example

$$M_1 = (\{q_0, q_1, q_2\}, \{Y, R\}, P_t, P_e)$$

$$P_t = \{ (q_0, q_1, 1), (q_1, q_1, 0.8), \\ (q_1, q_2, 0.15), (q_1, q_0, 0.05), \\ (q_2, q_2, 0.7), (q_2, q_1, 0.3) \}$$

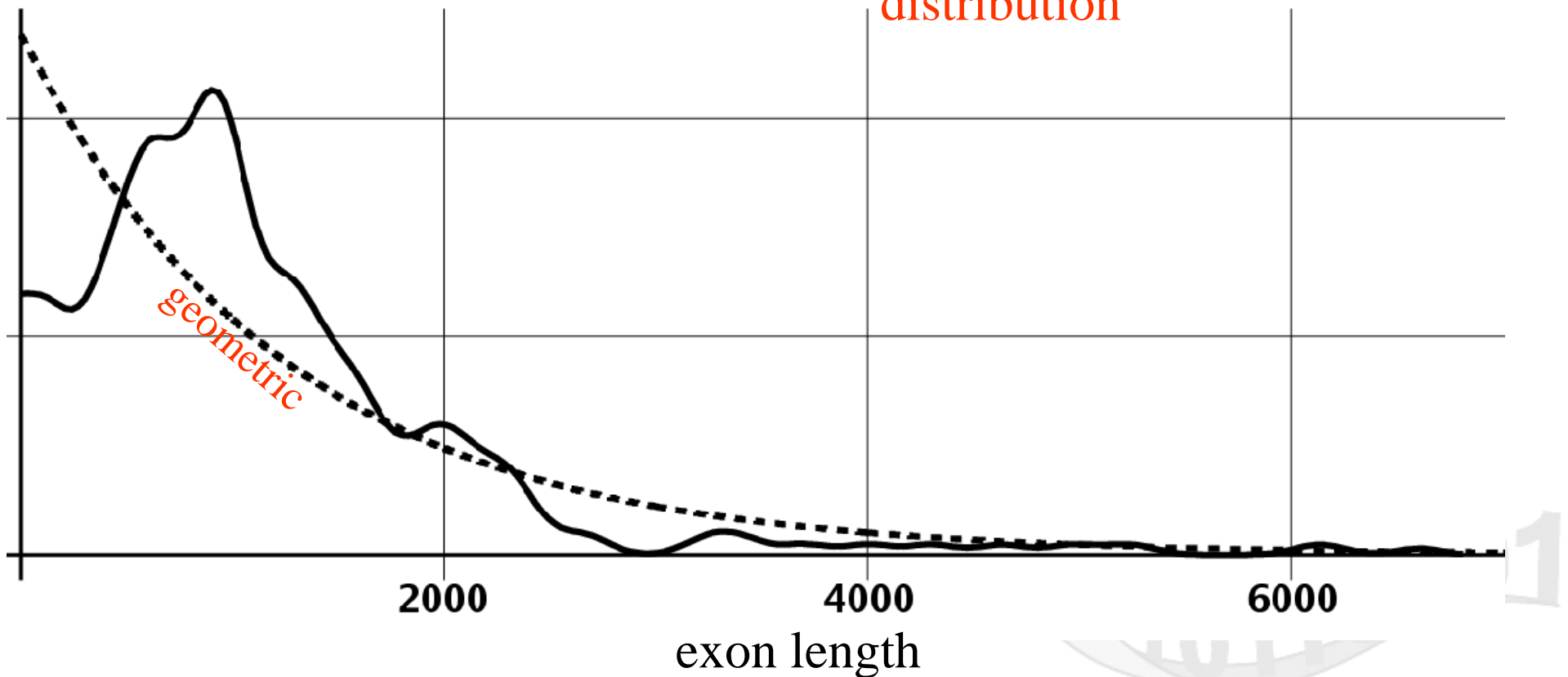
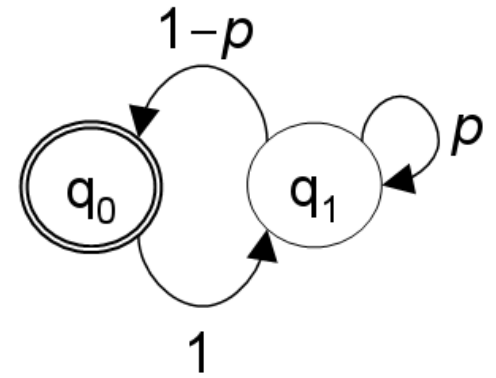
$$P_e = \{ (q_1, Y, 1), (q_1, R, 0), (q_2, Y, 0), (q_2, R, 1) \}$$



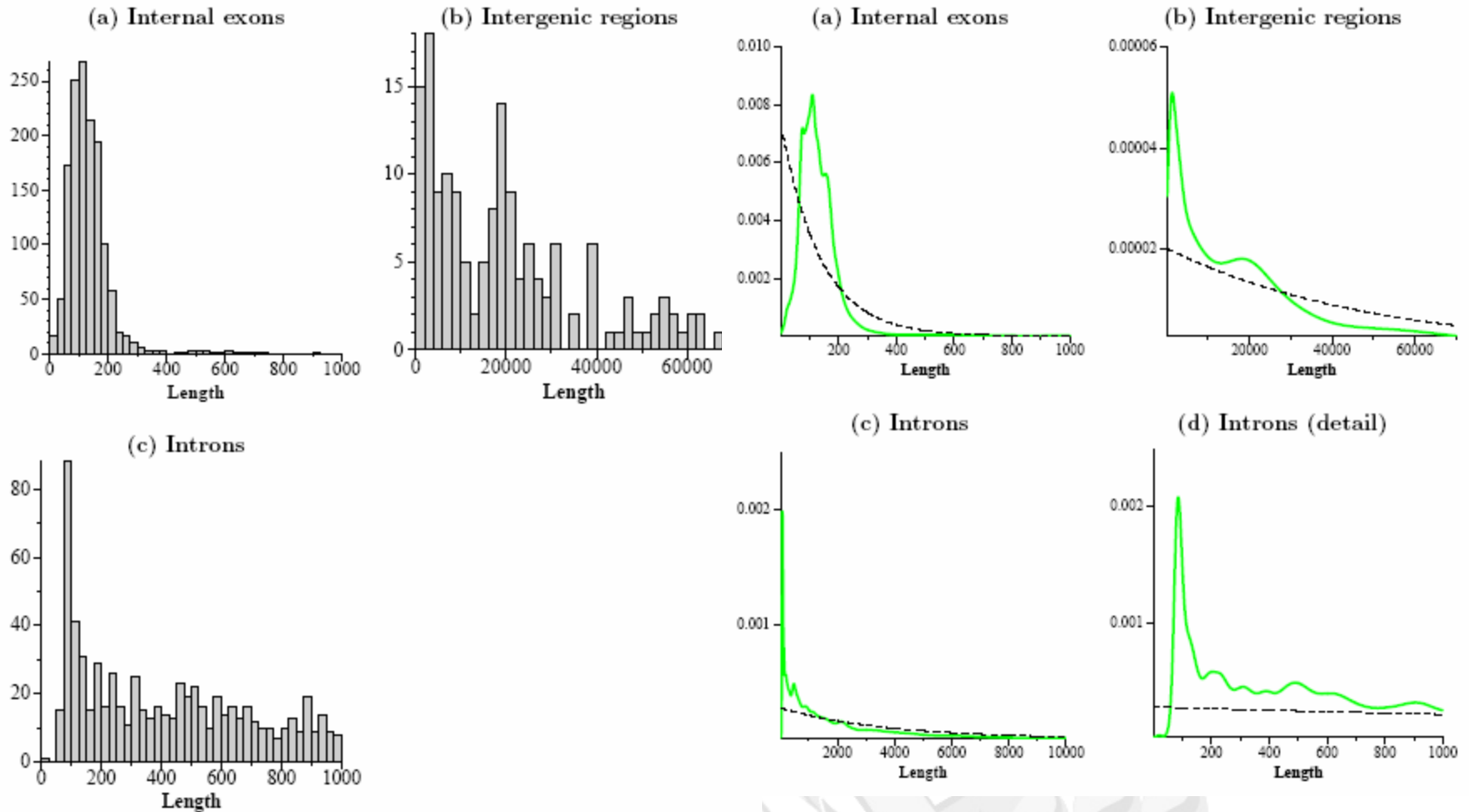
# HMMs & Geometric Feature Lengths

$$P(x_0 \dots x_{d-1} \mid \theta) = \left( \prod_{i=0}^{d-1} P_e(x_i \mid \theta) \right) p^{d-1} (1-p)$$

geometric distribution

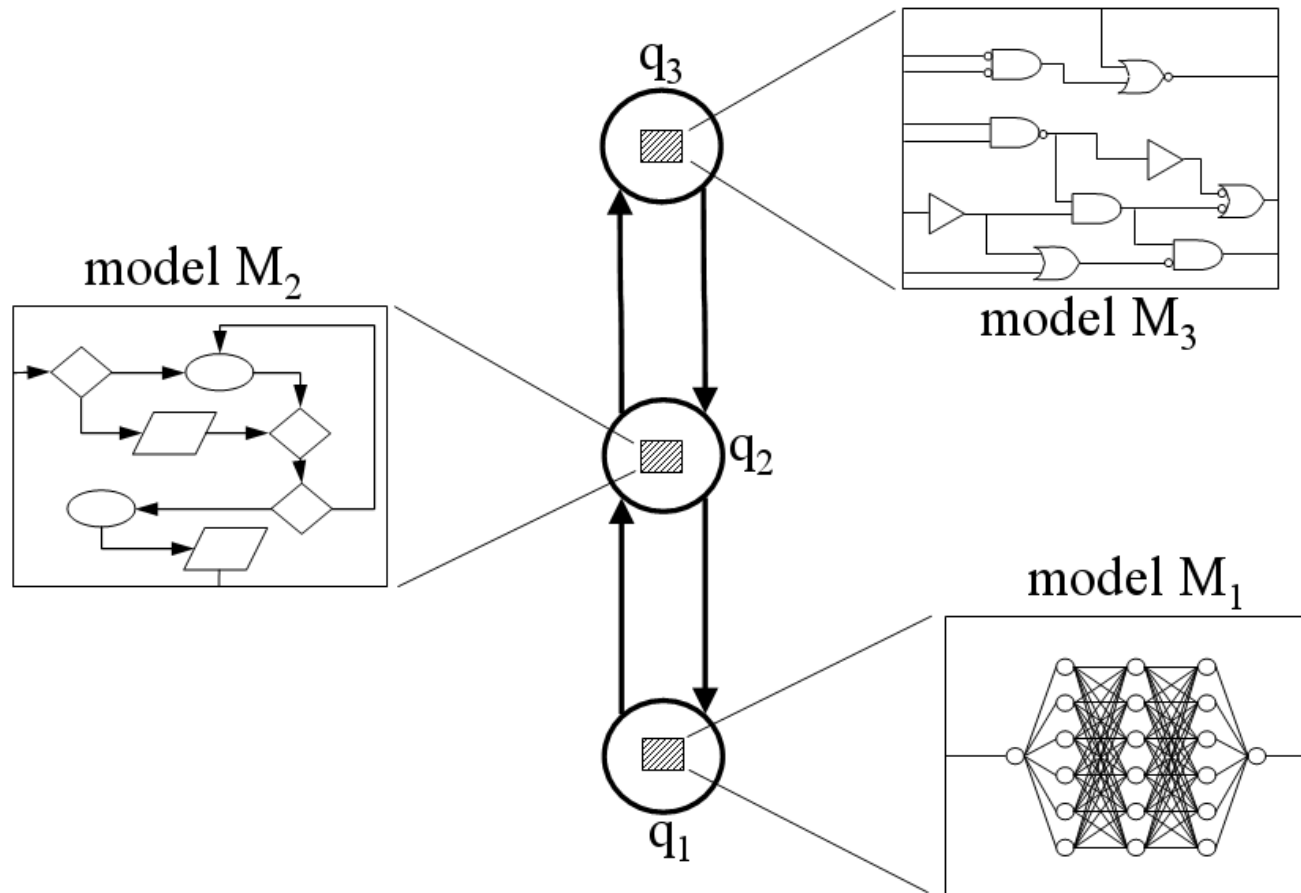


# Lengths Distribution in Human



Feature lengths were computed for Human chromosome 22 with RefSeq annotation (as of July 2005).

# Generalized Hidden Markov Models



## Advantages:

- \* Submodel abstraction
- \* Architectural simplicity
- \* State duration modeling

## Disadvantages:

- \* Decoding complexity

# Generalized HMMs

A GHMM is a stochastic machine  $M=(Q, \alpha, P_t, P_e, P_d)$  consisting of the following:

- a finite set of states,  $Q=\{q_0, q_1, \dots, q_m\}$
  - a finite alphabet  $\alpha=\{s_0, s_1, \dots, s_n\}$
  - a transition distribution  $P_t: Q \times Q \rightarrow [0,1]$  i.e.,  $P_t(q_j | q_i)$
  - an emission distribution  $P_e: Q \times \alpha^* \times \mathbf{N} \rightarrow [0,1]$  i.e.,  $P_e(s_j | q_i, d_j)$
  - a duration distribution  $P_d: Q \times \mathbf{N} \rightarrow [0,1]$  i.e.,  $P_d(d_j | q_i)$
- 

## Key Differences

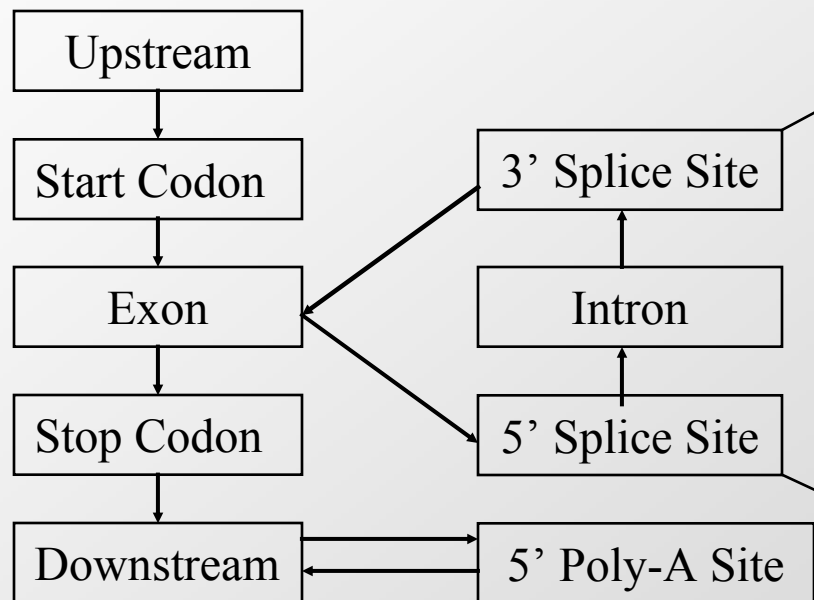
- each state now emits an entire subsequence rather than just one symbol
- feature lengths are now explicitly modeled, rather than implicitly geometric
- emission probabilities can now be modeled by any arbitrary probabilistic model
- there tend to be far **fewer states** => simplicity & ease of modification

## HMM-based Gene Finding

- GENSCAN (Burge 1997)
- FGENESH (Solovyev 1997)
- HMMgene (Krogh 1997)
- GENIE (Kulp 1996)
- GENMARK (Borodovsky & McIninch 1993)
- VEIL (Henderson, Salzberg, & Fasman 1997)

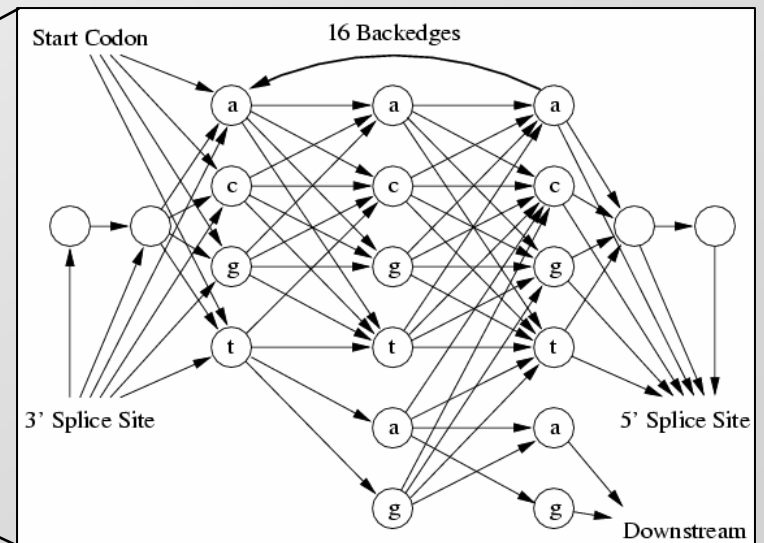
# VEIL: Viterbi Exon-Intron Locator

- Contains 9 hidden states or features
- Each state is a complex internal Markovian model of the feature
- Features:
  - Exons, introns, intergenic regions, splice sites, etc.



**VEIL Architecture**

## Exon HMM Model



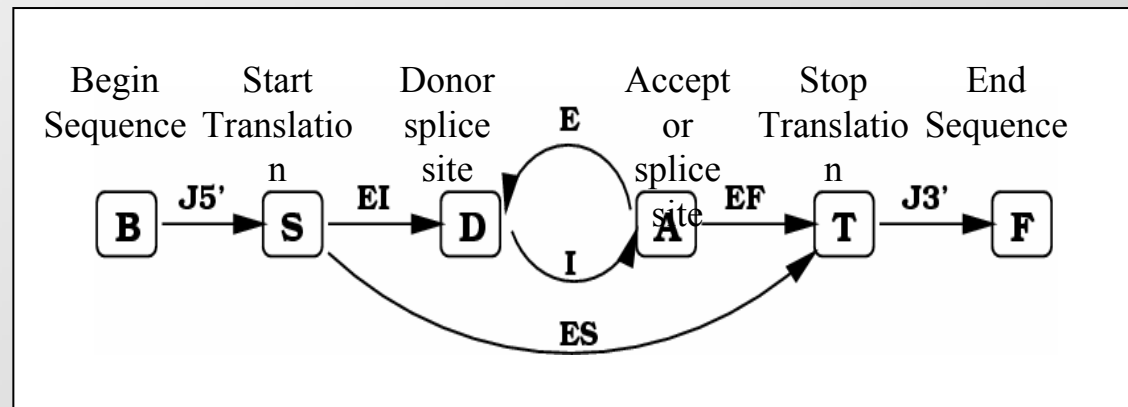
- Enter: start codon or intron (3' Splice Site)
- Exit: 5' Splice site or three stop codons (taa, tag, tga)



# Genie

- Uses a generalized HMM (GHMM)
- Edges in model are complete HMMs
- States can be any arbitrary program
- States are actually neural networks specially designed for signal finding

- J5' – 5' UTR
- EI – Initial Exon
- E – Exon, Internal Exon
- I – Intron
- EF – Final Exon
- ES – Single Exon
- J3' – 3'UTR



# Genscan Overview

- Developed by Chris Burge (Burge 1997), in the research group of Samuel Karlin, Dept of Mathematics, Stanford Univ.
- Characteristics:
  - Designed to predict complete gene structures
    - Introns and exons, Promoter sites, Polyadenylation signals
  - Incorporates:
    - Descriptions of transcriptional, translational and splicing signal
    - Length distributions (Explicit State Duration HMMs)
    - Compositional features of exons, introns, intergenic, C+G regions
  - Larger predictive scope
    - Deal w/ partial and complete genes
    - Multiple genes separated by intergenic DNA in a seq
    - Consistent sets of genes on either/both DNA strands
- Based on a general probabilistic model of genomic sequences composition and gene structure

# Genscan Architecture

- It is based on Generalized HMM (GHMM)
- Model both strands at once
  - Other models: Predict on one strand first, then on the other strand
  - Avoids prediction of overlapping genes on the two strands (rare)
- Each state may output a string of symbols (according to some probability distribution).
- Explicit intron/exon length modeling
- Special sensors for Cap-site and TATA-box
- Advanced splice site sensors

Image removed due to copyright restrictions.

**Fig. 3, Burge and Karlin 1997**

# GenScan States

- N - intergenic region
- P - promoter
- F - 5' untranslated region
- $E_{\text{sngl}}$  - single exon (intronless) (translation start -> stop codon)
- $E_{\text{init}}$  - initial exon (translation start -> donor splice site)
- $E_k$  - phase k internal exon (acceptor splice site -> donor splice site)
- $E_{\text{term}}$  - terminal exon (acceptor splice site -> stop codon)
- $I_k$  - phase k intron: 0 - between codons; 1 - after the first base of a codon; 2 - after the second base of a codon

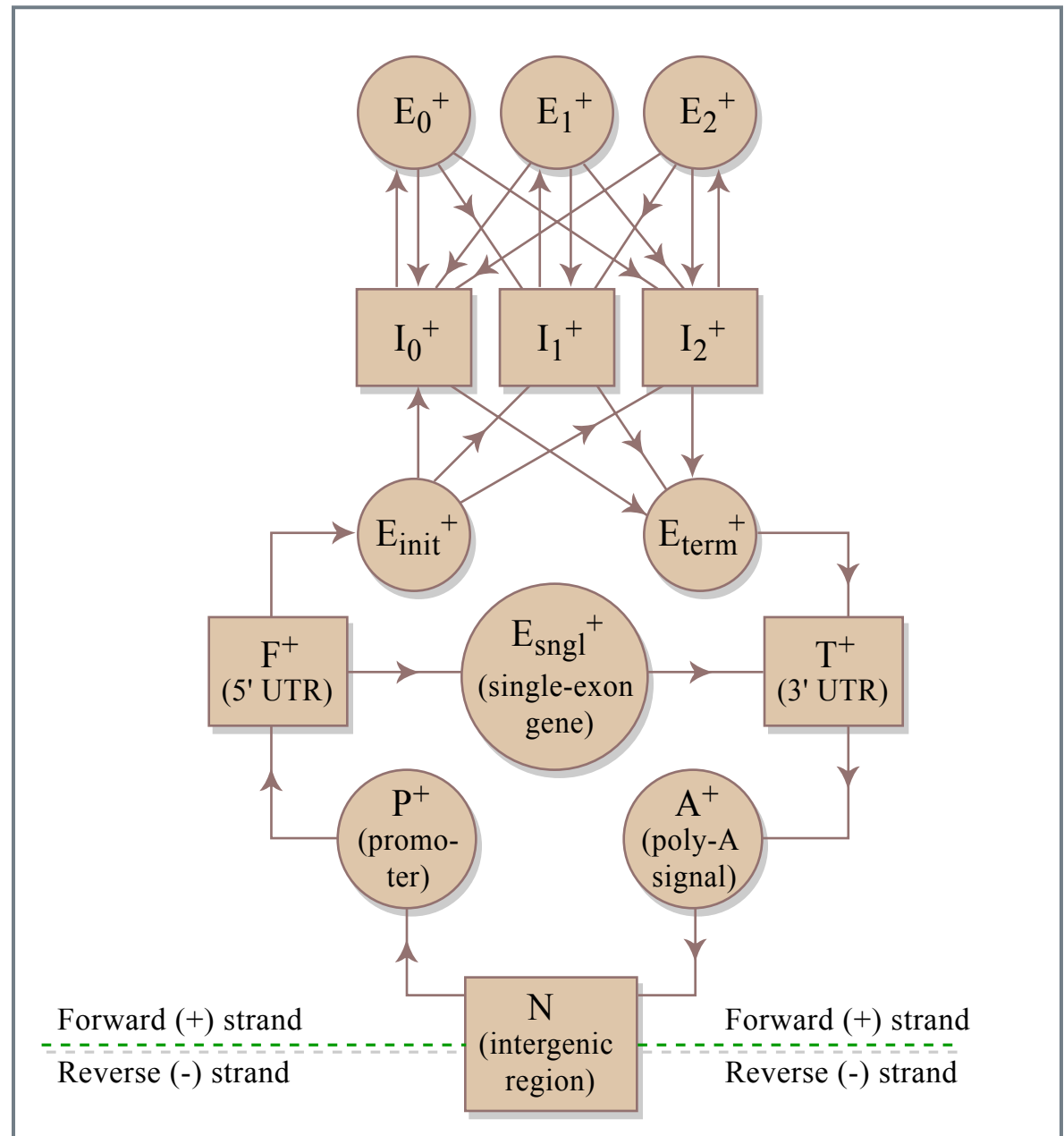


Figure by MIT OCW.

# Acknowledgement

- Slides are due to Manolis Kellis and William Majoros.