

# Motif Discovery Algorithms

Xiaohui S. Xie  
University of California, Irvine

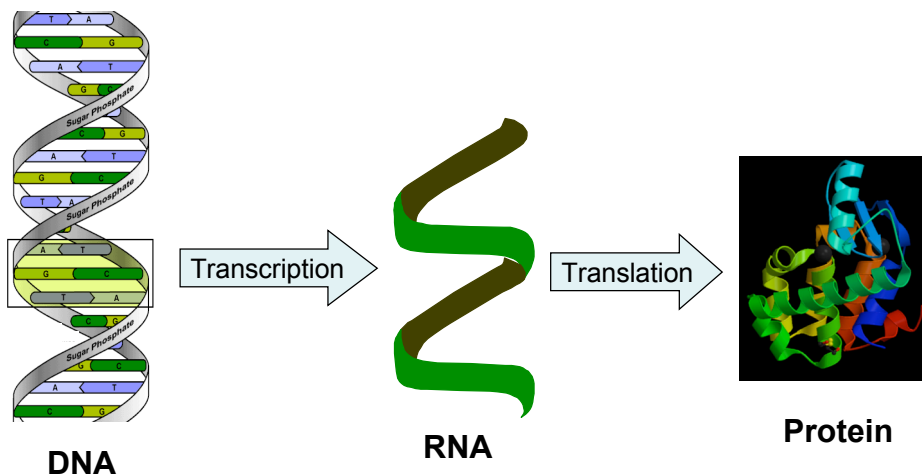
## Course Information Update

- Grading
  - 20% Homework
  - 10% Lecture scribe notes
  - 20% Midterm exam
  - 50% Final project
- Course Prerequisites:
  - Programming skill (Perl/Python, Matlab/R)
  - Statistics and Calculus

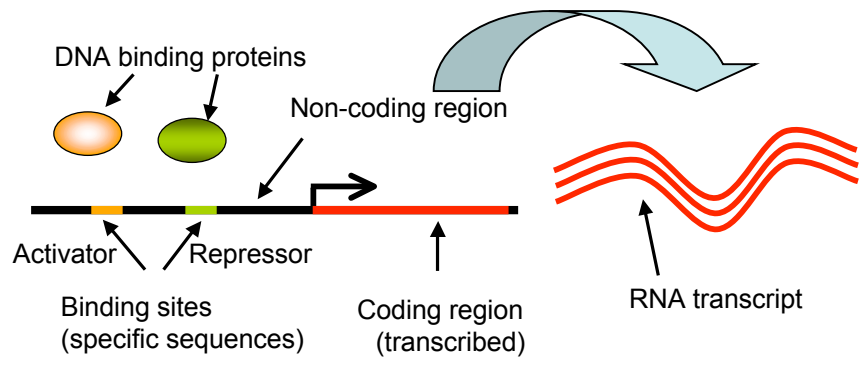
## Today's Goals

- Gene regulation
- Motif discovery algorithms
  - Enumeration
  - Statistical significance
  - Expectation-Maximization
  - Gibbs Sampling

## The Central Dogma

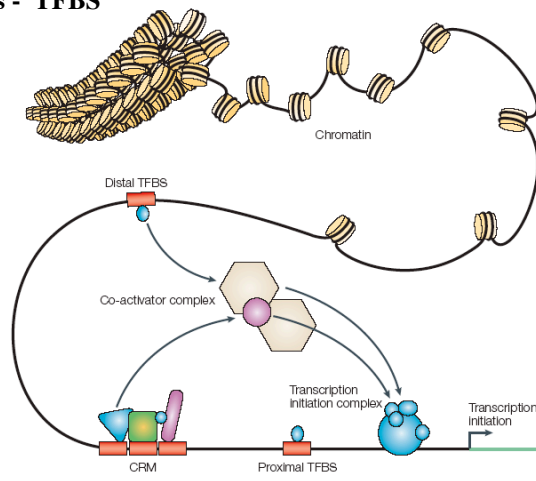


# Transcriptional Regulation



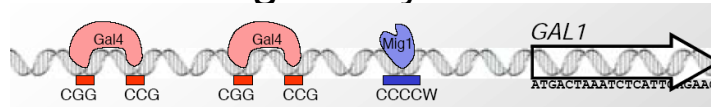
# Regulation in Eukaryotes

- Promotor
- Transcription Factors - TF
- Transcription Factor binding Sites - TFBS
- Cis-regulatory modules - CRM
- Transcription Start Site - TSS
- TATA boxes
- CG richness
- Phylogenetic Footprinting
- Combinatorial Interaction
- Enhancers



Wasserman and Sandelin (2004) 'Applied Bioinformatics for the Identification of Regulatory Elements' Nature Review Genetics 5.4.276

## Regulatory motifs



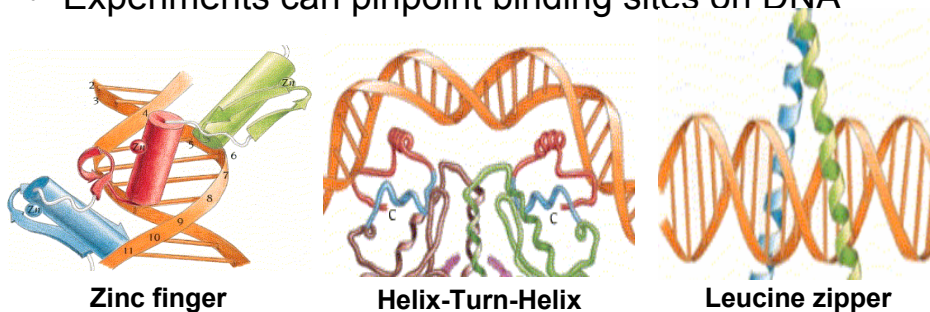
- Motifs are fundamental units of gene regulation:
  - **What** turns genes on (producing a protein) and off?
  - **When** is a gene turned on or off?
  - **Where** (in which cells) is a gene turned on?
  - **How many** copies of the gene product are produced?
- Specialized proteins (transcription factors) recognize these motifs

### What we know about regulatory motifs:

- Motifs are short (6-20 bp), sometimes degenerate
- Can contain any set of nucleotides (no ATG or other rules)
- Act at variable distances upstream (or downstream) from target gene (could be 100 Kb upstream or downstream)
- Human genome contains roughly 2000 motifs

## Transcription Factor Binding Sites

- Gene regulatory proteins contain structural elements that can “read” DNA sequence “motifs”
- The amino acid – DNA recognition is not straightforward
- Experiments can pinpoint binding sites on DNA



## Regulatory motif discovery

```
CAAACCTCTGCACGTGTCTCAAGGAATTTCCCGCCTCTGTCTTCTGAGTT
GGCTACAGATGTGTACCACGCACGTGGAACCCAGCTGATTTCCACCTTT
TTATCACGTGGAGCAAACGATTAGGGAGAATTAATTATCTCTTCCCTCTT
AGGAAATGATGTTTACCCTAACCCAAAATGTAAGACACGTGATTTATCAG
ACTACCTATAAAGAAGACACGTGAATCTGTCTGCTTGGTGGTGTAAAGGA
TTGGATTTGGTCCCACGTGATGTCAGAAGATTGCGGACCAAATCCCACTA
GCACACGTGGGGTCATTTGGAGAAAGATACTTTGTAAACATTGGACCTCTG
CATCTGTAAAACACGTGTGGGAATAGTAAGAATAATAATACTTGTCTCAC
ATGTGAAGGTAAAATGAGGTCATGCACGTGTGTGCACAGAATCTAGTCCA
AGAACATACCTGGCACTCAATTAATATGAGATAATTGTCCATGCCTTAA
GTATAAGATTTGTTATTACCGCACGTGTAAACACTACAGCATGAATTTC
ACTGCCAAAACACGTGTGGAGGTTTAAAGTTCTGATTCTGATGATGAAATA
CTCTGGCCTGCTACGTTAACACGTGAAAACAGCACTGATGGTAAAGGCTAA
TTGGATTTGGTCCCACGTGATGTCAGAAGATTGCGGACCAAATCCCACTA
ACTACCTATAAAGAAGACACGTGAATCTGTCTGCTTGGTGGTGTAAAGGA
```

Promoter sequences for 15 genes

## Method 1: Enumeration

List all potential motifs with a given length

For instance, 6-mer motifs

```
AAAAAA 0
AAAAAC 1
AAAAAG 2
AAAAAT 1
...
CACGTG 15
...
TTTTTT 1
TTTTTT 0
```

Total:  $4^6=4096$  6-mers

## Regulatory motif discovery

```
CAAACCTCTGCACGTGTCTCAAGGAATTTCCCGCCTCTGTCTTCTGAGTT
GGCTACAGATGTGTACCACGCACGTGGAAACCCAGCTGATTTCCACCTTT
TTATCACGTGGAGCAAACGATTAGGGAGAATTAATTATCTCTTCTCTT
AGGAAATGATGTTTACCCTAACCCAAAATGTAAGACACGTGATTTATCAG
ACTACCTATAAAGAAGACACGTGAATCTGTCTGCTTGGTGGTGTAAAGGA
TTGGATTTGGTCCACGTGATGTCAGAAGATTGCGGACCAAATCCCCTA
GCAACGTGGGGTCATTTGGAGAAAGATACTTTGTAACATTGGACCTCTG
CATCTGTAAAAACAGTGTGGGAATAGTAAGAATAATAACTTGTCTCAC
ATGTGAAGGTAAAATGAGGTCATGCACGTGTGTGCACAGAATCTAGTCCA
AGAACATACCTGGCACTCAATTAATATGAGATAATTGTCCATGCCTTAA
GTATAAGATTTGTTATTACCGCACGTGTAAACACTACAGCATGAATTTGC
ACTGCCAAAACAGTGTGGAGGTTTAAAGTTCTGATTCCTGATGATGAAATA
CTCTGGCCTGCTACGTAAACAGTGAACAGCACTGATGGTAAAGGCTAA
TTGGATTTGGTCCACGTGATGTCAGAAGATTGCGGACCAAATCCCCTA
ACTACCTATAAAGAAGACACGTGAATCTGTCTGCTTGGTGGTGTAAAGGA
```

Promoter sequences for 15 genes

## How to measure significance?

Suppose we observe that among the  $n$  promoter sequences, the motif occurs in  $k$  of them.

How surprised is the observation?

1. Curate a set of control sequences (total number:  $N$ ) that the motif is not enriched
2. Count the number of sequences that contain the motif ( $K$ )

## Representation of motifs, PWM

```

Site 1  A G A T G G A T G G
Site 2  T G A T T G A T G T
Site 3  T G A T G G A T G G
Site 4  A G A T T G A T C G
Site 5  T G A T G G A T T G
Site 6  T G A T G G A T T G
Site 7  A G A T G G A T T G
    
```

PWM represents frequencies of each base at each position in the motif \*

```

G      0 1.0  0 0  0.7 1.0  0  0  0.4 0.8
A      0.4 0  1.0 0  0  0  1.0 0  0  0
T      0.6 0  0 1.0 0.3 0  0  1.0 0.4 0.2
C      0  0  0  0  0  0  0  0  0.2 0
    
```

\* These days, PWM/PSSM can correspond to the frequency matrix or a likelihood matrix

## Positional weight matrix representation

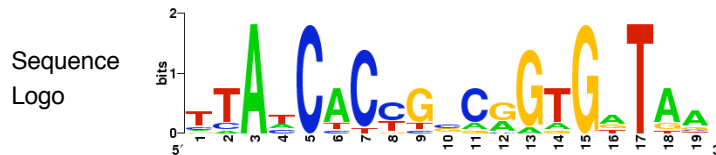
```

1  GTATCACC GCCAGTGGTAT
2  ATACCACTGGCGGTGATAC
3  TCAACACCCGCCAGAGATAA
4  TTATCTCTGGCGGTGTTGA
5  TTATCACC GCCAGATGGTTA
6  TAACCACTCTGCCGTGATAA
7  CTATCACC GCCAAGGGATAA
8  TTATCCCTTGCGGTGATAG
9  CTAACACCCGTCCGTGTTGA
10 TCAACACGCCACGGTGTAG
11 TTACCTCTGGCGGTGATAA
12 TTATCACC GCCAGAGGTAA
    
```

Lambda  
a  
cl/cro  
binding  
sites

$W_{ij}$

A:	9	9	94	25	1	71	1	1	1	1	9	17	32	9	17	1	48	1	71	63
C:	17	17	1	25	94	9	86	55	9	40	71	9	1	1	1	1	1	1	1	9
G:	9	1	1	1	1	1	9	71	40	9	55	86	9	9	4	25	1	17	17	
T:	63	71	1	48	1	17	9	32	17	9	1	1	1	71	1	25	4	9	9	

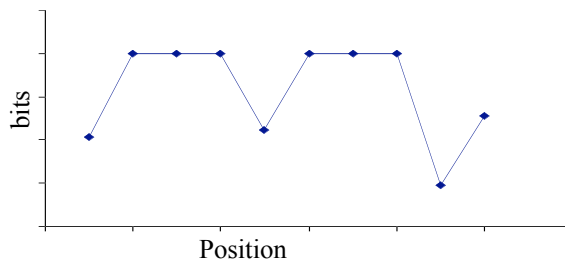


# Information content

The least variable positions likely are important for specifying the protein-DNA interaction  
Therefore high information content = low sequence variation at that position.

G	0	1.0	0	0	0.7	1.0	0	0	0.4	0.8	
A	0.4	0	1.0	0	0	0	1.0	0	0	0	
T	0.6	0	0	1.0	0.3	0	0	1.0	0.4	0.2	
C	0	0	0	0	0	0	0	0	0.2	0	
IC	1.0	2.0	2.0	2.0	1.1	2.0	2.0	2.0	0.5	1.3	= bit score of 15.9

Information Profile:



# Weight matrix, sequence logos

Corrected probabilities of observing a given nucleotide can be calculated using equation 1.

$$\text{Corrected probability calculation: } p(b, \hat{i}) = \frac{f_{b\hat{i}} + s(b)}{N + \sum_{b \in \{A,C,G,T\}} s(b)} \quad (1)$$

$f_{b\hat{i}}$  = counts of base  $b$  in position  $\hat{i}$ ;  $N$  = number of sites;  $p(b, \hat{i})$  = corrected probability of base  $b$  in position  $\hat{i}$ ;  $s(b)$  = pseudocount function

A position weight matrix (PWM) is constructed by dividing the nucleotide probabilities in (1) by expected background probabilities and converting the values to a log-scale (see equation 2).

$$\text{PWM conversion: } W_{b\hat{i}} = \log_2 \frac{p(b, \hat{i})}{p(b)} \quad (2)$$

$p(b)$  = background probability of base  $b$ ;  $p(b, \hat{i})$  = corrected probability of base  $b$  in position  $\hat{i}$ ;  $W_{b\hat{i}}$  = PWM value of base  $b$  in position  $\hat{i}$

The quantitative PWM score for a putative site is the sum of the PWM values for each nucleotide in the site (see equation 3)

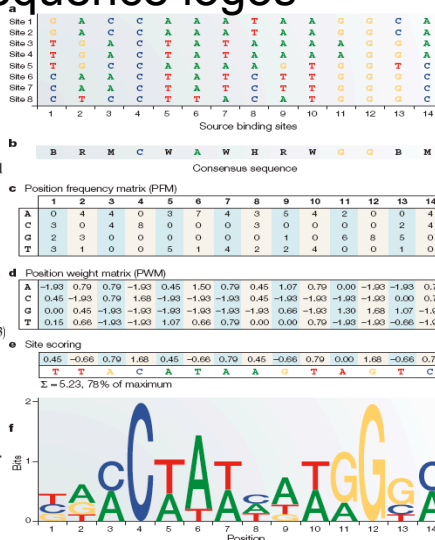
$$\text{Evaluation of sequences: } S = \sum_{i=1}^w W_{b_i} \quad (3)$$

$l_i$  = the nucleotide in position  $i$  in an input sequence;  $S$  = PWM score of a sequence;  $w$  = width of the PWM

Probability values (1) can be used to determine the total information content (in bits) in each position (see equation 4).

$$\text{Information content calculation: } D_i = -2 + \sum_b p_{b\hat{i}} \log_2 p_{b\hat{i}} \quad (4)$$

$D_i$  = information content in position  $\hat{i}$ ;  $p_{b\hat{i}}$  = corrected probability of base  $b$  in position  $\hat{i}$



**Very high frequency of false positives. A model for binding of MyoD will yield 10<sup>6</sup> binding sites, while only 10<sup>3</sup> might be real.**

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