Multihop performance of geographic random forwarding for ad hoc and sensor networks

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***Abstract*— In this paper, we study a novel forwarding technique based on geographical location of the nodes involved and random selection of the relaying node via contention among receivers. We focus on the multihop performance of such a solution, in terms of average number of hops to reach a destination as a function of the distance and of the average number of available neighbors. An idealized scheme (in which the best relay node is always chosen) is discussed, and its performance is evaluated by means of both simulation and analytical techniques. A practical scheme to select one of the best relays is shown to achieve performance very close to that of the ideal case.**

I. Introduction

Ad hoc and sensor networks are recently attracting a lot of interest in the research community. One of the main issues here is the necessity to save energy, since the nodes are usually battery powered and need to be alive for a relatively long time. A few papers have recently appeared which propose MAC, routing, and topology maintenance schemes that try to save energy based on aggressive power-off strategies. In fact, it has been recognized that the only way a node can save substantial energy is by powering off the radio, since transmitting, receiving and listening to an idle channel are functions which require a comparable amount of power. As a consequence of this key observation, MAC and routing strategies need to be revisited since, for example, CSMA-based access schemes need all nodes to continuously listen to the channel while, on the other hand, nodes which power off their radio may end up not being reachable and/or aware of activity in the network. The main problem in this scenario is therefore that of combining protocols which minimize the amount of time the radio is on with effective strategies for MAC and routing.

An example of how topology can be maintained in the presence of sleeping nodes is provided by SPAN in [1], where the authors propose that in a dense network several disjoint sets of nodes be identified, each able to guarantee connectivity and bandwidth to all nodes. As long as one of these sets is active at any given time, the network is connected; on the other hand, since when one set is active the others can sleep, the percentage of time a node must be active is drastically reduced.

STEM [2] provides a way to establish communications in the presence of sleeping nodes. Each sleeping node wakes up periodically to listen. If a node wants to establish communications, it starts sending out beacons polling a specific user. Within a bounded time, the polled node will wake up and receive the poll, after which the two nodes are able to communicate. An interesting feature of STEM is that a dual radio setup is envisioned, with separate frequencies used for wakeup and actual data transmission.

GAF [3] is similar to SPAN in a sense, since it envisions the use of only a fraction of the nodes at any given time. The specific approach of GAF is to divide the area in square regions, called grids, in such a way that any two nodes in neighboring grids are within range of each other. With this provision, grids can be treated as equivalent (or virtual) nodes, in the sense that all nodes in the same grid can be interchangeably used for routing purposes. The price to pay for this guarantee is that the hop length is significantly smaller than the radio range (by a factor of $\sqrt{5}$, which is the largest possible distance between two nodes in adjacent grids [3]). This may result in inefficiency in terms of latency and energy consumption (more hops than possibly needed).

As to MAC schemes, most papers in the literature assume either TDMA-based schemes [4], or multi-channel setups in which parallel transmissions can be performed without interference [5], [6], or variants of classic contention-based schemes, usually based on RTS/CTS handshake in order to mitigate the hidden terminal problem [7], [8]. Past work on geographical routing includes [9], [10].

A common characteristic of the above schemes is that, at the MAC layer and often also at the routing layer, when a node decides to transmit a packet (as the originator or a relay) it specifies the MAC address of the neighbor to which the packet is being sent. Knowledge of the network topology (though in many cases only local in extent) is required since a node needs to know its neighbors and possibly some more information related to the availability of routes to the intended destination. This topological information can be acquired at the price of some signaling traffic, and becomes more and more difficult to maintain in the presence of network dynamics (e.g., nodes which move or turn off without coordination).

We propose here Geographic Random Forwarding (GeRaF, pronounced as "giraffe"), a novel transmission scheme based on geographical routing where packets are relayed on a best-effort basis, i.e., the actual node which acts as a relay is not known a priori by the sender, but rather is decided after the transmission has taken place. This idea leverages on the fact that in the wireless environment broadcast is free (from the sender's point of view) and that in the presence of randomly changing topologies a node may not be aware of which of its current neighbors is in the best position to act as a relay. In a sense, this is like doing contention at the receiver's end, which is untraditional because in classic schemes it is the transmitter which contends for the channel. Here, since the intended recipient is not specified, multiple nodes may be able to receive the packet, and a *receiver contention scheme* is therefore needed to guarantee that a single relay is chosen, thereby avoiding packet duplication.

In this paper, we focus on the multihop performance of such a solution, in terms of average number of hops to reach a destination as a function of the distance and of the average number of available neighbors. An idealized scheme (in which the best relay node is always chosen) is discussed, and its performance is evaluated by means of both simulation and analytical techniques. A practical scheme to select one of the best relays is shown to achieve performance very close to that of the ideal case.

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II. Basic idea

We assume that each node has some knowledge of its own position and of the position of the sink node, i.e., the node where the information needs to be delivered. For simplicity of description, we assume that this location information is perfect (robustness to erroneous location information is discussed in [11]) and that propagation can be characterized in terms of coverage radius (that is, two nodes are neighbors if they are within the coverage radius of each other). While this is certainly a crude model for propagation, it is assumed here as a first step towards understanding fundamental behaviors.

The basic idea is the following. Once a node has a packet to send, it sends a message using some type of broadcast address while specifying its own location and the location of the intended destination. All active (listening) nodes in the coverage area will receive this message and will assess their own priority in trying to act as a relay, based on how close they are to the destination. More details about an actual protocol based on collision avoidance can be found in in [12].

Suppose a mechanism is in place to make sure that the relaying node is in fact the one closest to the destination. The MAC/routing scheme then continues similarly. The relayed packet is addressed to a broadcast address and contains the location of the transmitter and of the final destination, thereby providing a means to geographically route it without any routing tables or topological information (except for the location of the destination). Notice that the scenario we have in mind is one in which sensor nodes may be stationary and densely deployed, and randomly turn on and off, thereby providing a random topology. If the density of active nodes is appropriate, it is likely that the node closest to the destination will be almost the best possible, i.e., will provide an advancement towards the destination close to the coverage radius. Notice that in this case there is no need for sleeping nodes to coordinate (they turn on and off randomly). Also, since no attempt is made to gain topology information, the frequency at which nodes turn on and off may be fairly high, which provides for small latencies.

Notice also that the fact that we do not address a specific node allows us to use one of the first available nodes within the coverage area, as opposed to STEM in which we have to wait for a specific node to wake up. In this case, we can easily decrease the duty cycle of each node without increasing latency if adequate node density is available. In fact, the rate at which any of N nodes wakes up is N times that of each single node, and therefore we can maintain similar network connectivity while saving more energy if we increase N and decrease the wakeup rate by keeping their product constant.

It should be observed at this point that GAF [3] tries to achieve a similar objective, by making all nodes in each grid interchangeable from a routing perspective. However, a major weakness of GAF's approach is precisely the requirement that this routing feature be guaranteed, which forces hops to cover less than half the distance allowed by the radio range.

III. Analysis of the idealized scheme

Consider the following simple scenario. A source wants to deliver a packet to a destination, in the absence of cross traffic (which corresponds to the case in which the network is mostly monitoring, and occasionally a message is generated). Nodes are randomly placed in the region according to a Poisson process with density ρ' nodes per unit area. Due to the use of sleep modes, each node is available as a relay with probability d, so that the density of actually available nodes is $d\rho$. Let the radio range be normalized to 1, and let D be the distance between the source and the destination. Finally, let $M = d\rho\pi$ be the average number of available relays in the coverage area.

We initially assume ideal operation, whereby the neighbor closest to the destination (if any) is in fact selected as the relaying node, unless the destination itself is within range, in which case the packet is directly delivered. We are interested in computing the number n of hops necessary to reach the destination as a function of the distance D and of the density of active nodes $d\rho$. Clearly, it must be $n \geq D$, and we expect that $\lim_{d\rho\to\infty} n = \lfloor D+1 \rfloor$.

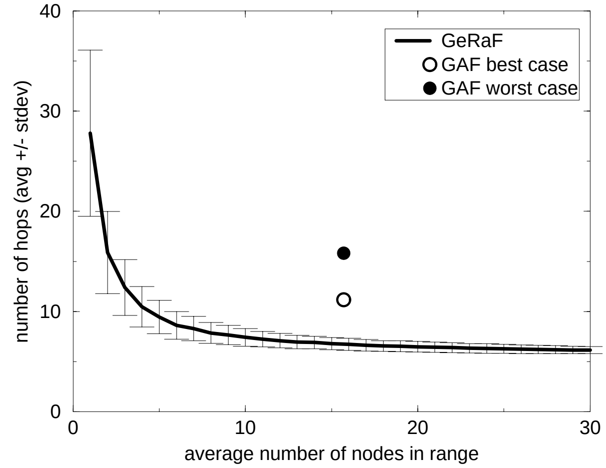

Fig. 1. Average number of hops $\pm$ standard deviation vs. average number of active neighbors. Distance $D = 5$.

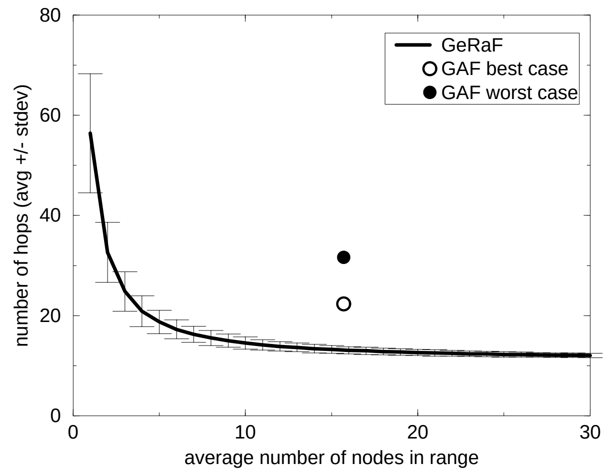

Fig. 2. Average number of hops $\pm$ standard deviation vs. average number of active neighbors. Distance $D = 10$.

We set up a simple simulation to evaluate the number of hops which are necessary to reach the destination. We assume first that the transmit node is at distance D from the final destination. We distribute randomly a Poisson distributed number of relays (with average M) in the coverage area and, among them, we select the one that is closest to the destination, which in turn becomes the transmit node for the next hop. This step is repeated (using the new distance to the destination instead of D) until a relay within range of the destination is reached, from which a single hop is needed. If it happens that there are no nodes in range which are closer to the destination than the transmit node, one hop is counted but the position of the transmit node is not updated. This corresponds to the fact that, if no relays are present to provide an advancement towards the destination, a transmit node would try again, and in the next attempt the set of possible relays is independently generated so that with probability one the packet arrives to the destination in a finite amount of time.

The observed behavior is shown in Figures 1 through 3, where simulation results for the average number of hops (with

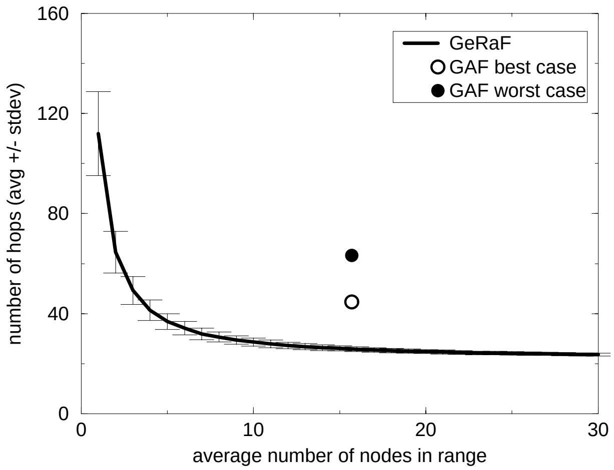

Fig. 3. Average number of hops $\pm$ standard deviation vs. average number of active neighbors. Distance $D = 20$.

bars denoting the standard deviation) are plotted versus the node density, expressed in terms of the average number of active neighbors, M. The expected behavior is observed. Interestingly, the results for different distances scale proportionally to the distance itself. While this is intuitive, the fact that the scaling is almost exactly proportional was not entirely obvious.

In GAF, which is the only other scheme with which we can compare directly, the area is divided into square grids of side $r = 1/\sqrt{5}$ (normalized to radio coverage). Notice that if we consider all nodes in a grid as a single equivalent node located at its center, each hop corresponds to a distance of exactly r. This distance is not necessarily the advancement towards the destination, as this depends on the relative orientation of the source-destination direction and the grid layout. In the best case (they are parallel), one hop leads to an advancement of r units toward the destination, in the worst case (they form a 45^o angle) the one-hop advancement is $r/\sqrt{2} = 1/\sqrt{10}$ (which is three times shorter than allowed by the radio range).

Regarding active node density, GAF requires that at least one node be present in each grid, which corresponds to an average node density of at least one in each $1/5$ square unit area, i.e., 5π nodes within radio range. The isolated points shown in the graph correspond to the best-case and worst-case performance of GAF.

Notice that the curve for our optimal scheme lies well below and to the left of GAF's points, which means that for comparable active node density (i.e., overall energy consumption due to sleep duty cycle) we can deliver a packet in much fewer hops (which is an effect of GAF's underutilization of radio coverage) or, equivalently, for a given number of hops to the destination (which corresponds to latency as well as transmission energy used to deliver a packet) we need much fewer nodes within range, and this corresponds to a reduced duty cycle given the same density of physical nodes. Note that the points corresponding to GAF do not take into account details such as synchronization and coordination issues among nodes and the possibility of some grids being empty, and therefore are to be seen as somewhat optimistic estimates of GAF's performance. The curves for our scheme are themselves idealized, but as shown later a practical scheme would perform almost as well.

A. Simple analytical bounds on the multihop performance of GeRaF

In this section, we approach the problem of evaluating the multihop performance of random forwarding using analytical

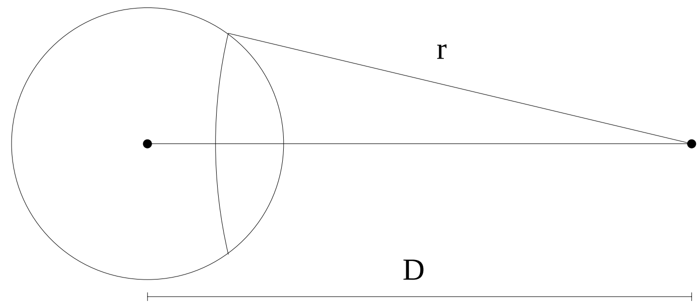

Fig. 4. Circle intersection for the analysis

techniques, for the ideal case in which the best possible relay is selected.

Consider the case in which a node needs to send a packet to a destination which is at distance D (all distances are normalized to the coverage radius, which is therefore taken to be unity). Let γ be the remaining distance after one hop, which must satisfy $D-1 \le \gamma \le D$. The probability distribution of γ depends on the area of the intersection between two circles with centers at distance D and with radii 1 and r, respectively. More specifically, the probability that the remaining distance γ is at least r is the probability that the intersection of the two circles contains no relays (see Figure 4). If $A(r, D)$ is the area of the intersection, we have

$$P[\gamma \ge r] = e^{-MA(r,D)/\pi}, D-1 \le r < D \quad (1)$$
$$P[\gamma = D] = e^{-MA(D,D)/\pi} \quad (2)$$

where we account for the fact that if the whole relay region is empty, there is no advancement and the remaining distance does not change.

By geometric considerations, the area in question is found as

$$\begin{aligned} A(r,D) &= 2\int_{D-1}^{r} a \arccos\left(\frac{a^2+D^2-1}{2aD}\right) da \\ &= \beta(w,1) + \beta(D-w,r) \end{aligned} \quad (3)$$

in which $w = \frac{D^2-r^2+1}{2D}$ and

$$\beta(x,y) = \frac{1}{2}\left(\pi - 2\arcsin\frac{x}{y} - 2\frac{x}{y}\sqrt{1-\left(\frac{x}{y}\right)^2}\right) \quad (4)$$

The advancement towards the destination is $\zeta = D - \gamma$, with probability distribution

$$P[\zeta \le a] = \begin{cases} e^{-MA(D-a,D)/\pi} & 0 \le a \le 1 \\ 0 & a < 0 \\ 1 & a > 1 \end{cases} \quad (5)$$

Let $f_\zeta(a) = f_\zeta^c(a) + P[\zeta = 0]\delta(a)$ be the pdf of the advancement, where $f_\zeta^c(a)$ is the derivative of $P[\zeta \le a]$ in $a \in (0,1)$. The average advancement is then found as [11]

$$E[\zeta] = \int_0^1 a f_\zeta(a) da = 1 - \int_0^1 e^{-MA(D-a,D)/\pi} da \quad (6)$$

Notice that $E[\zeta]$ depends on D. More precisely, it can be shown that it is an increasing function of D [11].

It is possible to show that the average number of hops which is needed to reach a destination at distance D can be bounded as (details are in [11])

$$\frac{D-1}{E[\zeta(D)]} + 1 \le E[n] < \frac{D}{E[\zeta(1)]} + 1 \quad (7)$$

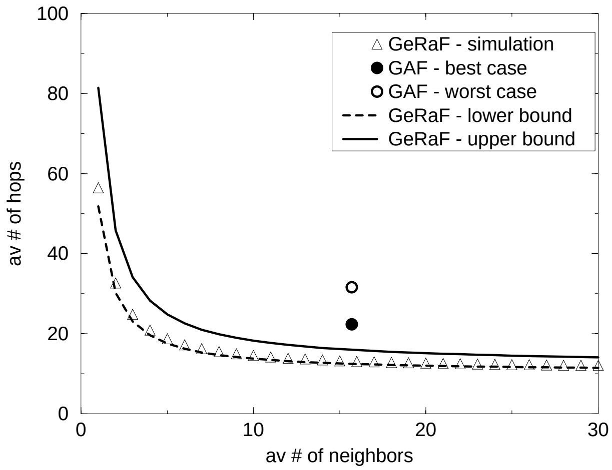

Fig. 5. Average number of hops vs. average number of active neighbors. Simulation and analytical bounds. Distance $D = 10$.

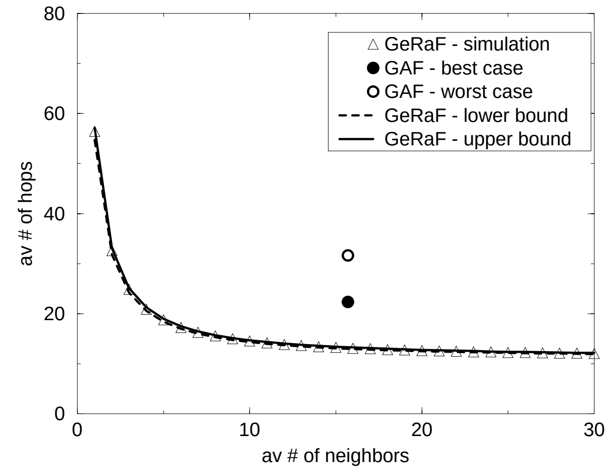

Fig. 6. Average number of hops vs. average number of active neighbors. Simulation and analytical bounds (recursion). Distance $D = 10$.

where $E[\zeta(D)]$ is given in (6).

In Figure 5, these bounds are compared with the simulation results for the case $D = 10$. It can be seen that, even though the upper bound is fairly loose in general, the lower bound is an excellent approximation. Similar results were found for the other cases.

B. Recursive evaluation of the multihop performance of GeRaF

As another approach, which is more accurate but also more computation-intensive, we consider here the recursive computation of the average number of hops to reach a destination at distance D.

We quantize the whole range of possible distances from the destination, from 0 through D. Let ν be the number of quantization intervals per unit distance, so that the total number of intervals considered is $D\nu$ (for analytical convenience, we assume that D contains an integer number of such quantization intervals). More specifically, let $\Delta_i = \left(\frac{i-1}{\nu}, \frac{i}{\nu}\right]$ be the i-th quantization interval.

If the transmitter is within coverage of the final destination, i.e., in $\Delta_i, i = 1, \ldots, \nu$, then the number of hops is one. Consider the case in which the transmitter is at distance $\gamma = i/\nu > 1$. The probability that the advancement will lead to a remaining distance in interval $i - \nu + k$ is

$$\omega(i,k) = \begin{cases} e^{-A\left(\frac{i-\nu+k-1}{\nu}, \frac{i}{\nu}\right)} - e^{-A\left(\frac{i-\nu+k}{\nu}, \frac{i}{\nu}\right)} & k = 1, \ldots, \nu \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

In fact, going from $\gamma = i/\nu$ to a point in $\left(\frac{j-1}{\nu}, \frac{j}{\nu}\right]$ corresponds to having no relays with residual distance $\gamma \leq \frac{j-1}{\nu}$ but at least one relay with $\gamma \leq \frac{j}{\nu}$. In addition, with probability $\omega_0(i) = e^{-A\left(\frac{i}{\nu}, \frac{i}{\nu}\right)}$ there is no advancement.

Consider first a pessimistic bound, in which whenever the relay is in interval j the remaining distance is set to the maximum possible, i.e., j/ν. In this case, we can write the following recursive relationship for the upper bound on the average number of hops, $n_1(i)$, from position i

$$n_1(i) = 1 + \omega_0(i) n_1(i) + \sum_{k=1}^{\nu} \omega(i,k) n_1(i - \nu + k) \tag{9}$$

from which we obtain

$$n_1(i) = (1 - \omega_0(i) - \omega(i,\nu))^{-1} \sum_{k=1}^{\nu-1} \omega(i,k) n_1(i - \nu + k) \tag{10}$$

Similarly, we can make the optimistic assumption that whenever the relay is in interval j the remaining distance is set to the minimum possible, i.e., $(j-1)/\nu$. In this case, we can write the following recursive relationship for the lower bound on the average number of hops, $n_2(i)$, from position i

$$n_2(i) = 1 + \omega_0(i) n_2(i) + \sum_{k=1}^{\nu} \omega(i,k) n_2(i - \nu + k - 1) \tag{11}$$

from which we obtain

$$n_2(i) = (1 - \omega_0(i))^{-1} \sum_{k=1}^{\nu} \omega(i,k) n_2(i - \nu + k - 1) \tag{12}$$

The average number of hops $n(D)$ to reach a destination at distance D (assumed to be equal to an integer number of quantization intervals) can then be bounded as follows

$$n_2(D\nu) \leq n(D) \leq n_1(D\nu) \tag{13}$$

The tightness of these bounds improves as ν increases and $\lim_{\nu \to \infty} n_1(D\nu) = \lim_{\nu \to \infty} n_2(D\nu) = n(D)$. As an example, Figure 6 shows these bounds for $\nu = 50$ and $D = 10$.

IV. A PRACTICAL SCHEME

In a previous subsection, we have evaluated the statistics of the number of hops which are needed to deliver a packet to the destination. In that simple computation, we assumed that the active neighbor closest to the destination was selected. Here, we propose a simple implementable mechanism to make this selection, and show that almost ideal performance can be achieved.

A node who wants to transmit a packet broadcasts a message. All active nodes in its coverage area will hear the message. Each node will then determine its own distance from the final destination and assess its own adequacy as a relay. This is done by first dividing the coverage area in two parts: the relay region, which contains all points closer to the destination than the transmitting node, and the non-relay region, which contains all other points within range. Nodes in the non-relay region are never selected as relays.

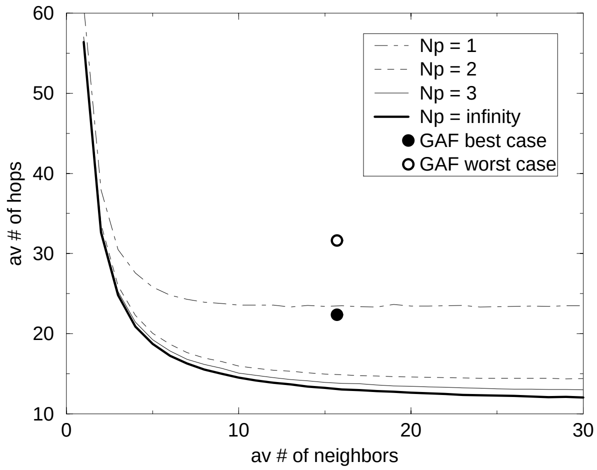

Fig. 7. Average number of hops vs. average number of active neighbors for $N_p = 1, 2, 3, 4$ and ∞. Distance $D = 10$.

Further, the relay region is sliced in N_p "priority regions," based on the distance from the destination. More specifically, region $\mathcal{A}_i$ contains all points in the coverage region whose distance from the final destination is $D - 1 + (i-1)/N_p \leq \gamma < D - 1 + i/N_p, i = 1, 2, \ldots, N_p$ (D is the distance from the transmitting node to the final destination and the radius of coverage is normalized to unity).

Nodes in $\mathcal{A}_1$, which are those closest to the destination, contend first. If no nodes are found in $\mathcal{A}_1$, then nodes in $\mathcal{A}_2$ contend, and so on until either nodes are found or all regions are empty. In the latter case, one "zero-hop" is counted and the transmitter retries (note that since nodes wake up randomly, each reattempt will see an independent set of nodes if the reattempt times are appropriately chosen with respect to the on/off activity characteristics of the nodes).

The specific details of the contention scheme are not important for our purposes. An example of a possible scheme is the following. Assume that time is slotted. In the first slot after the transmitter has sent a message, all nodes which received the message and are in $\mathcal{A}_1$ send a reply. If only one reply is generated, the contention successfully terminates. If no reply is generated, then $\mathcal{A}_1$ was empty and the transmitter will send a signaling message which will then trigger all nodes in $\mathcal{A}_2$ to send a reply, and so on. If multiple nodes reply simultaneously, the collision can be resolved by means of any collision resolution scheme (e.g., a splitting algorithm). One such algorithm is considered in detail in [12].

Our simulation is therefore organized as follows. The packet starts at the origin node. Within the coverage area, a number of active nodes are randomly placed, and subdivided into regions according to their geographical location with respect to the destination. The non-empty region which is closest to the destination is then selected, and one of the nodes in it is randomly picked and chosen as the relay. Note in fact that if the collision resolution scheme within a priority region does not use geographical information (which on the other hand is used in associating nodes to the regions), all nodes in a region are equally likely to win the collision. One hop is counted, and everything is repeated using the relay node as the origin. If the relay node is within range of the destination, the process terminates in one more hop. If all regions are empty, one hop is counted and the origin node is not updated.

The result of this procedure as a function of the number of regions considered is shown in Figure 7, where the average number of hops is plotted vs. the average number of active nodes within coverage. The case $N_p = 1$ corresponds to the fact that any node closer to the destination than the origin will do. This of course does not perform well, but as soon as at least two regions are considered the performance is close to the case $N_p = \infty$, which corresponds to the idealized scheme presented before. These results show that a simple rule in which each node determines its own region and then contends with all other nodes in that region is essentially as good as the (practically infeasible) ideal selection as long as $N_p \geq 2$. Clearly the value of N_p and the specific strategy with which contention is resolved will affect the time it takes to transmit a packet, and therefore is relevant for latency as well as energy consumption [12]. However, since here we are merely interested in hop count, this simple evaluation is adequate.

V. Conclusions

In this paper, we have described a novel forwarding technique based on geographical location of the nodes involved and random selection of the relaying node via contention among receivers. We first focused on the multihop performance of such a solution, in terms of average number of hops to reach a destination as a function of the distance and of the average number of available neighbors. An idealized scheme (in which the best relay node is always chosen) was discussed, and its performance was evaluated by means of both simulation and analytical techniques. A practical scheme to select one of the best relays was shown to achieve performance very close to that of the ideal case. More details and discussion about the multihop behavior of GeRaF can be found in [11], whereas the details of an implementable protocol based on this idea as well as its energy and latency performance can be found in [12].

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