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Test IV

Version A

ICS 6D
Winter 2017
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Instructions

- Wait until instructed to turn over the cover page.
- There are 25 questions, each worth 1 point. There is no partial credit.
- Unless otherwise instructed, you can leave your answer as an arithmetic expression, including the $P(n, k)$, $n!$, or $\binom{n}{k}$ notation. You will get full credit as long as the expression evaluates to the correct answer. You can not use "..." in your answers.
- Circle your answer to each question. You should have only one expression inside the circle for each question. We will score the question based only on the value of the expression inside the circle.
- **Important:** There are questions on both sides of the page. The back of the last page is for scratch work.

1. A set of 30 different tasks are assigned to a set of 10 different processors. Each task is assigned to exactly one processor. A processor can be assigned more than one task. Since the tasks are different, it matters which task is assigned to which processor. How many ways are there to distribute the tasks to the processors if there is no restriction on the number of tasks that can go to a processor?

2. Below is a 6-subset (a subset with 6 elements) chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Give the 6-subset that comes next in lexicogrpahical order:

$$\{2, 4, 5, 8, 9, 10\}$$

3. Here is a statement about two sets: in lexicographic order $\{2, 1, 3, 4\} < \{1, 32, 140, 4\}$. Is the statement true or false?

4. The 7^{th} row of Pascal's triangle is: 1 7 21 35 35 21 7 1.
   Give the 8^{th} row of Pascal's triangle.

5. 20 identical prizes are distributed to members of an audience in a game show. There are 220 members in the audience. How many different ways are there to distribute the prizes so that no audience member receives more than one prize?

6. How many strings of length 11 over the alphabet $\{a, b, c\}$ start with "aa" or end with "aa" or both?

7. A set of 30 different tasks are assigned to a set of 10 different processors. Each task is assigned to exactly one processor. A processor can be assigned more than one task. Since the tasks are different, it matters which task is assigned to which processor. How many ways are there to distribute the tasks to the processors so that each processor is assigned the same number of tasks?

8. 20 identical prizes are distributed to members of an audience in a game show. There are 220 members in the audience. How many ways are there to distribute the prizes if there are no restrictions on the number of prizes that a particular audience member can receive?

9. Felicity goes to a grocery store to purchase 13 candy bars. The store sells 8 varieties of candy bars. There is a large number of each variety of candy bar available except for Snickers bars. The store only has 4 Snickers bars available. How many ways are there for Felicity to make her selection?

10. How many solutions are there to the equation: $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 37$, where each x_i is an integer that satisfies $x_i \geq 2$?

11. Below is a permutation of the set $\{1, 2, 3, 4, 5, 6\}$. Give the permutation that comes next in lexicogrpahical order:
$$(4, 6, 2, 5, 3, 1)$$

12. Gretchen goes to buy a dozen donuts from a donut store that sells five varieties of donuts. One of the varieties of donuts sold is chocolate. How many ways are there to select the donuts if she must have **exactly** one chocolate donut in her selection?

13. 40 identical tasks are distributed to 10 different processors. Each task is assigned to exactly one processor and the processors must all get exactly the same number of tasks. How many ways are there to distribute the tasks to the processors?

14. Fred is an administrative assistant at a company and must place an order for 25 box lunches for a board meeting. The restaurant where he places the order sells 10 varieties of box lunches. The variety "Turkey Club Sandwich" is the most popular selection, so Fred makes sure there are at least five of that variety in his selecion. How many ways are there for Fred to make his selection? Note that we are not concerned about who will get which lunch, just the number of each variety in the order.

15. There are 34 singers in a school choir. 18 of the singers are women and the other 16 are men. The choir director must select 12 singers from the choir to be in the honors choir. How many ways are there to select the honors choir if there must be at least one man and at least one woman in the choir?

16. 12 employees are each assigned to an office. There are four offices and each can hold three people. The offices are different, so it matters who gets assigned to which office. How many different ways are there to make the assignment?

17. A red die and a blue die are thrown. How many outcomes are there in which the red die comes up with an even number or the blue die comes up with an even number or both?

18. 20 different prizes are distributed to members of an audience in a game show. There are 220 members in the audience. How many ways are there to distribute the prizes so that no audience member receives more than one prize?

19. How many strings of length 12 over the alphabet $\{a, b, c, d, e\}$ have exactly three a's or exactly three b's or both?

20. Here is a statement about two 4-tuples: in lexicgraphic order $(2, 1, 3, 4) < (1, 32, 140, 4)$. Is the statement true or false?

21. What is the coefficient of the $c^{10}d^3$ term in $(4c - 5d)^{13}$?

22. The Binomial Theorem says that for any positive integer n and any real numbers x and y, $\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n$. Use the Binomial Theorem to determine the value for $\sum_{k=0}^{n} (-2)^k \binom{n}{k}$. Your answer should not be a sum of terms.

23. A fair coin is flipped twice. Since the coin is fair, each outcome is equally likely. What is the probability that at least one of the flips comes up heads?

24. How many binary strings of length 6 have a sequence of at least 5 consecutive 1's somewhere in the string?

25. How many ways are there to permute the letters in "ABRACADABRA"?

This area is for scratch work.