Part I: Select the statements to correctly fill in the blanks of the proof of the Theorem 1 given below.

Theorem 1. *For any real number x, if $x^2 - 6x + 5 \geq 0$, then $x \geq 5$ or $x \leq 1$. (Inclusive "or").*

Proof: Proof by contrapositive.

Assume that __________(a)__________.

We will prove that __________(b)__________.

By assumption, we know that __________(c)__________. By subtracting 1 from both sides of the inequality, we get that $x - 1 > 0$ and therefore $x - 1$ is positive.

By assumption, we know that __________(d)__________. By subtracting 5 from both sides of the inequality, we get that $x - 5 < 0$ and therefore $x - 5$ is negative.

The product of a positive number and a negative number is negative. Therefore, __________(e)__________.

Multiplying out the left side of the inequality, we get that __________(f)__________.

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1. In the proof of the Theorem 1, what expression should go in the space labeled (a)?

   A. $x < 5$ and $x > 1$.
   C. $x < 5$ or $x > 1$
   B. $x^2 - 6x + 5 \geq 0$
   D. $x \geq 5$ or $x \leq 1$

2. In the proof of the Theorem 1, what expression should go in the space labeled (b)?

   A. $x^2 - 6x + 5 < 0$
   C. $x \geq 5$ or $x \leq 1$
   B. $x < 5$ and $x > 1$.
   D. $x^2 - 6x + 5 \geq 0$

3. In the proof of the Theorem 1, what expression should go in the space labeled (c)?

   A. $x < 5$
   C. $x^2 - 6x + 5 \geq 0$
   B. $x > 1$
   D. $x \geq 1$

4. In the proof of the Theorem 1, what expression should go in the space labeled (d)?

   A. $x^2 - 6x + 5 \geq 0$
   C. $x \geq 1$
   B. $x > 1$
   D. $x < 5$

5. In the proof of the Theorem 1, what expression should go in the space labeled (e)?

   A. $(x - 1)(x - 5) \geq 0$
   C. $x^2 - 6x + 5 \geq 0$
   B. $(x - 1)(x - 5) < 0$
   D. $(x + 1)(x - 5) < 0$

6. In the proof of the Theorem 1, what expression should go in the space labeled (f)?

   A. $x^2 - x - 5 < 0$
   C. $x^2 - 6x + 5 \leq 0$
   B. $x^2 - 6x + 5 \geq 0$
   D. $x^2 - 6x + 5 < 0$

Part II: For the next four questions, your choices will be one of the following five statements. Any of the statements could be the correct answer for more than one question.

A. x is rational and y is rational.

B. $x + y$ is rational.

C. x is irrational and y is irrational.

D. x is irrational or y is irrational.

E. $x + y$ is irrational.

Theorem 2. *For any two real numbers, x and y, if x is rational and y is rational then $x + y$ is rational.*

Statement 1: A direct proof of Theorem 2 would assume for real numbers x and y that __(+)__and would prove that __(*)__.

7. What expression should go in the space labeled (+) in Statement 1? A

8. What expression should go in the space labeled (*) in Statement 1? B

Statement 2: A proof by contraspositive of Theorem 2 would assume for real numbers x and y that __(&)__and would prove that __($)__.

9. What expression should go in the space labeled (&) in Statement 2? E

10. What expression should go in the space labeled ($) in Statement 2? D

Part III:
Define the following sets:

- $A = \{x \in \mathbb{Z} : x \text{ is a multiple of } 3\}$
- $B = \{3, 5, 7, 9\}$.
- $C = \{2, 3, 4, 5\}$.

Indicate whether the following statements are true or false. Select A for true and B for false. The universe set is the set of all integers.

11. $|B| = |C|$ True

12. $|A \cap B| = |A \cap C|$ False

13. $A \cap C \subseteq A \cap B$ True

14. $C - B \subseteq \overline{A}$. True

15. $B \cup C = \{3, 5\}$. False

16. $2 \in A \cup C$. True

17. $\{2, 3\} \in C$ False

18. $\{3\} \in P(C)$ True

19. $\overline{A} \cap B \cap C = \emptyset$ False

20. $\emptyset \in A$. False.