

1. Fill in the steps of the argument below to show that  $(\neg p \rightarrow q) \wedge \neg p \equiv q \wedge \neg p$ . Each step is labeled with the name of a propositional law. Fill in each blank with a proposition so that each proposition can be obtained from the one before it by applying the stated law.

$(\neg p \rightarrow q) \wedge \neg p$	Start
	Conditional Identity
	Involution law
	Distributive law
	Complement law
$q \wedge \neg p$	Identity law

2. The domain of discourse is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates:

- $P(x, y)$ :  $x$  knows  $y$ 's phone number. (*A person may or may not know their own phone number*).
- $D(x)$ :  $x$  missed the deadline.
- $N(x)$ :  $x$  is a new employee.

Give a logical expression for each of the following sentences:

- (a) Someone knows everyone's phone number.
- (b) Everyone knows someone's phone number.
- (c) There is at least one new employee who missed the deadline.
- (d) Sam knows the phone number of everyone who missed the deadline.

3. This problem refers to the predicates and domain defined in the previous question. Consider a situation in which there are five people in the group. The table below gives values for the predicates  $D$  and  $N$  for each member of the group. For example, Bert did not miss the deadline because the truth value in the row labeled Bert and the column labeled  $D(x)$  is F. Using these values, circle the quantified expressions that evaluate to true.

Name	$D(x)$	$N(x)$
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

- (a)  $\forall x(D(x) \vee N(x))$
- (b)  $\exists x((x \neq \text{Al}) \rightarrow N(x))$
- (c)  $\exists x(\neg D(x) \wedge \neg N(x))$
- (d)  $\exists x(\neg D(x) \rightarrow \neg N(x))$
- (e)  $N(\text{Bert}) \rightarrow D(\text{Bert})$
- (f)  $\forall x(\neg N(x) \rightarrow D(x))$
- (g)  $\forall x((x \neq \text{Sam}) \rightarrow N(x))$
- (h)  $\forall x(D(x) \rightarrow N(x))$
- (i)  $\exists x(D(x) \leftrightarrow N(x))$
- (j)  $D(\text{Sam}) \wedge N(\text{Sam})$