CS 262: Computational Complexity **Due: March 18, 2010**

Homework 7

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Do four of the following five problems:

1. **FP** is the set of functions from $\{0,1\}^*$ to $\{0,1\}^*$ that can be computed by a deterministic Turing Machine in polynomial time. Show that computing the permanent of a matrix with integer entries can be done in $\mathbf{FP}^{\mathbf{\#SAT}}$. Note that the integer entries may be negative but you will get partial credit if you prove this under the restricion of non-negative entries.

2. Define a language L to be *downward self-reducible* if there's a polynomial-time algorithm R that for any n and $x \in \{0,1\}^n$, $R^{L_{n-1}}(x) = L(x)$ where by L_k we denote an oracle that solves L on inputs of size at most k. Prove that if L is downward-self-reducible, then $L \in$ PSPACE.

3. A *strong* non-deterministic Turing Machine is one that has three possible outcomes: "yes", "no" and "maybe". We say that such a machine decides a language L if the following is true: whenever $x \in L$, then all computations end up with "yes" or "maybe" and at least one ends up with "yes". If $x \notin L$, then all computations end up with "no" or "maybe" and at least one ends up with "no". Show that if L is decided by a strong non-deterministic machine in polynomial time then $L \in \mathbf{NP} \cap \mathbf{co} - \mathbf{NP}$.

4. Prove that every language L in **NL** that is not the empty set or $\{0,1\}^*$ is complete for **NL** under polynomial time reductions.

5. Recall the definition of QSAT:

$$\text{QSAT } = \{\Phi(x_1, \ldots, x_n) \mid \exists x_1 \forall x_2 \ldots \forall x_n \Phi(x_1, \ldots, x_n) = 1\},$$

   where Φ is a 3-CNF formula. Show that $P^{QSAT} = NP^{QSAT}$.