

Homework 3

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1. Suppose L_1 and L_2 are in $\mathbf{NP} \cap \mathbf{co-NP}$. Then show that $L_1 \oplus L_2$ is in $\mathbf{NP} \cap \mathbf{co-NP}$, where $L_1 \oplus L_2 = \{x \mid x \text{ is in exactly one of } L_1 \text{ and } L_2\}$.
2. Prove that if *every* unary language in \mathbf{NP} is also in \mathbf{P} , then $\mathbf{EXP} = \mathbf{NEXP}$. Recall that a language is unary if and only if it is a subset of 1^* .
3. Say that class C_1 is *superior* to C_2 if there is a language L_1 in C_1 such that for every language $L_2 \in C_2$ and every n sufficiently large, there is an input whose length is between n and n^2 on which L_1 and L_2 differ. That is, there is a string x whose length is between n and n^2 such that either $x \in L_1$ and $x \notin L_2$ or $x \notin L_1$ and $x \in L_2$.
 - (a) Is $\mathbf{DTIME}(n^4)$ superior to $\mathbf{DTIME}(n)$?
 - (b) Why does our proof of the Non-deterministic Time Hierarchy not prove that $\mathbf{NTIME}(n^{1.5})$ is superior to $\mathbf{NTIME}(n)$?