Homework 2

Due: January 29, 2010

1. In class, we showed that NP can be defined to be the class of languages L such that there exists a polynomial-time decidable language R where

$$L = \{x \mid \exists y, |y| \leq |x|^k, (x, y) \in R\}.$$

   Is it possible to define an analogous definition of the class NEXP that does not use the notion of a non-deterministic Turing Machine? Why or why not?

2. Show that if $f(n) \geq n$ and $g(n) \geq n$ are proper complexity functions then $\text{TIME}(f(n)) = \text{NTIME}(f(n))$ implies that $\text{TIME}(f(g(n))) = \text{NTIME}(f(g(n)))$.

3. Define a *coding* κ to be a mapping from Σ to Σ. Note that κ need not be one-to-one. If $x = \sigma_1 \ldots \sigma_n$, where each $\sigma_i \in \Sigma$, then we define $\kappa(x) = \kappa(\sigma_1) \ldots \kappa(\sigma_n)$. If L is a language, then $\kappa(L)$ is defined to be $\{\kappa(x) | x \in L\}$.

   (a) Prove that NP is closed under codings. That is, show that if $L \in \text{NP}$ and κ is a coding defined on the alphabet of L, then $\kappa(L) \in \text{NP}$.

   (b) We expect that P is not closed under codings, but we can not prove this without establishing that $\text{P} \neq \text{NP}$. Instead, show that P is closed under codings if and only if $\text{P} = \text{NP}$.