CS 262: Computational Complexity *Instructor: Sandy Irani*

Homework 1

Due: January 15, 2010

1. The definition of 3SAT allows for clauses to have fewer than three literals. We will define a resticted version of 3SAT called EXACT-3SAT in which the literals in each clause must be distinct and every clause must have exactly three literals. (Recall that a *literal* is a variable or the negation of a variable). Prove that 3SAT reduces to EXACT-3SAT.

2. The *Kleene closure* of a set L is the set $L^* = \{x_1x_2\cdots x_k : k \geq 0; x_1, x_2, \ldots, x_k \in L\}$. Notice that the notation Σ^* is compatible with this notation. Show that PSPACE is closed under Kleene closure.

3. A complexity class C is said to be *closed under reductions* if whenever L reduces to L' and $L' \in \mathcal{C}$, then $L \in \mathcal{C}$. Prove that P and PSPACE are closed under reductions. Is TIME(n^2) closed under reductions? Justify your answer.

4. Suppose that a Turing Machine can insert and delete symbols from its tape in addition to overwriting them.

   (a) Define carefully the transition function and computation of such machines.

   (b) Show that such a machine can be simulated by an ordinary Turing Machine with a quadratic loss of efficiency.