

Computational Resources:

- running time.
- memory / space.
- parallelism.
- randomness.
- rounds of interaction.

Decidability: unlimited resources.

Big Questions:

<u>Conj</u> No	$\stackrel{Q}{=} P \stackrel{?}{=} NP$	finding a solution $\equiv$ verifying a solution.
No	$P \stackrel{?}{=} NC$	can every alg be parallelized?
No	$P \stackrel{?}{=} L$	can every efficient alg use a small amt of memory
No	$EXP \stackrel{?}{\subseteq} P/poly$	are there small circuits to solve all every prob that uses exp time.
YES	$P \stackrel{?}{=} BPP.$	can every rand alg be de-randomized?

Encoding:

ex: Graph s, t .

$\Sigma = \{ () , ; , 0, 1, \dots, 9 \}$

#vertices: $(-, -) (-, -) (-, -)$

No unary encoding.

N in binary $\lg(N)$ bits.
in unary N characters.

Given an encoding: computational problem.

$f: \Sigma^* \rightarrow \Sigma^*$

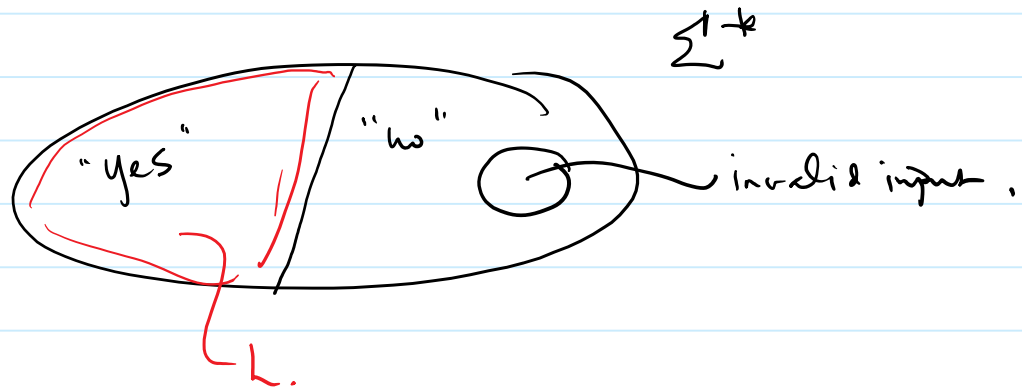
Decision problem: $f: \Sigma^* \rightarrow \{\text{yes}, \text{no}\}.$

Search: Input: int N .
Output: prime factorization of N .

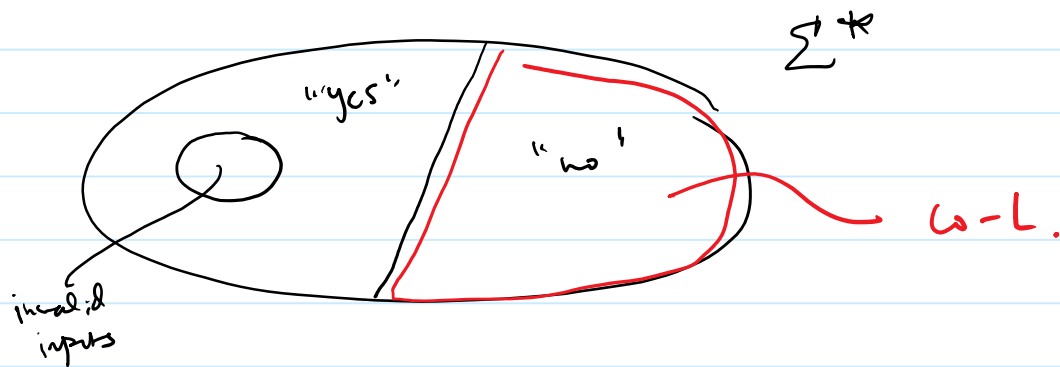
Decision: Input: int N, k
Output: does N have a factor $\leq k$.

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Language L : subset of Σ^* that correspond to "yes" instances.



$\text{co-}L$ "complement of L " $\Sigma^* - L$.

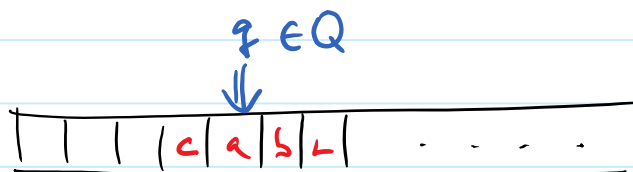


Complexity class: Set of languages that can be computed w/ limited resource.

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- Complexity class:
- contain natural/important probs.
 - capture some computational resource.
 - Complete problems are natural + important.
 - robust under variations in the computational model.
 - closed under $\wedge, \vee, \neg$

Turing Machine

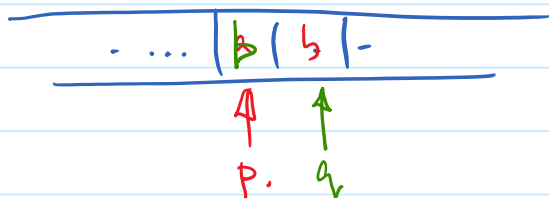


Σ = finite alphabet.
must contain $\sqcup$

Q : set of states (finite)
 Σ : finite alphabet.
 q_{start} q_{acc} q_{rej}

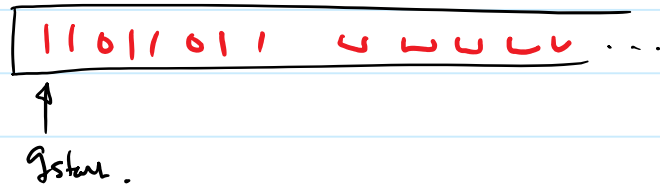
transition function: $\delta : Q \times \Sigma \rightarrow Q \times \Sigma \times \{L, R, \sqcup\}$

$$\delta(p, a) = (q, b, R)$$



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Start conf's:



Halts if q_{acc} or q_{rej} is reached

- 3 ways to compute:
- (1) function comp.
output $f(x)$ left in tape
when TM halts.
 - (2) language decision:
TM reaches q_{acc} if $x \in L$.
TM reaches q_{rej} if $x \notin L$.
 - (3) language recognition
TM reaches q_{acc} if $x \in L$.
TM reaches q_{rej} or
never halts if $x \notin L$.