Instructions: Leave your answer for the questions below as an arithmetic expression, including the $P(n, k)$ or $\binom{n}{k}$ notation. You do not have to compute a final numeric value. If a question has multiple parts, the restrictions from one part do not apply to the other parts.

1. *(6 points)* A family goes to the animal shelter to select three pet cats. The shelter currently has 27 cats. At the shelter there are 10 long-haired cats and 17 short-haired cats.

   (a) How many ways are there for the familiy to select their cats?

   (b) How many ways are there for the family to make their selection if they want two long-haired cats and one short-haired cat?

   (c) How many ways can they make their selection if they want at least one long-haired cat?

2. *(5 points)* A company is placing an order for boxed lunches for a meeting of its executive board. There are 10 members of the board and each will have one lunch. The company is getting the lunches from a restaurant that has 25 varieties of boxed lunches.

   (a) How many different ways are there for the company to place the lunch order for the 10 lunches? Note that it is not important who gets what lunch. All that matters is how many of each of the 25 possible varieties are purchased.

   (b) How many ways are there for the company to place the lunch order if the 10 lunches purchased are all different? (Again, who gets what lunch is not important, just how many of each lunch are purchased).

3. *(12 points)* How many strings are over the alphabet $\{a, b, c, d\}$ have length 8 and the following properties:

   (a) No other restrictions.

   (b) Start with an a.

   (c) Exactly three a's.

   (d) Have the same number of each character.

   (e) Have at least one c?

4. *(4 points)* 100 pianists compete in a piano competition.

   (a) In the first round, 25 of the 100 are selected to go on to the next round. How many different outcomes are there for the first round?

   (b) In the second round, the judges select a first, second, third, fourth and fifth place winners of the competition from among the 25 pianists who advanced to the second round. How many outcomes are there for the second round of the competition?

5. *(10 points)* A teacher distributes 10 identical homework passes to her class of 20 students. How many ways are there for her to distribute the passes if:

   (a) Each student gets at most one homework pass.

   (b) There are no restrictions on the number of homework passes a student can get.

   (c) There are no restrictions on the number of homework passes a student can get, except that one particular student, Sam, gets at least two homework passes.

   (d) There are no restrictions on the number of homework passes a student can get, except that one particular student, Sam, gets at most two homework passes.

6. *(4 points)* Frank decides to distribute his comic book collection to his four nephews. There are 100 comic books in the collection and each comic book in the collection is different.

   (a) How many ways are there for him to distribute the comic books if there are no restrictions on how many go to each nephew?

   (b) How many ways are there for him to distribute his collection to the four nephews if he decides that each nephew should get exactly the same number of comic books?

7. *(4 points)* A standard playing deck has 52 distinct cards. How many ways are there to deal hands from a standard playing deck to four players if

   (a) Each player gets exactly 13 cards.

   (b) Each player gets seven cards and the rest of the cards remain in the deck?

8. *(5 points)* A class of 20 kids line up for recess.

   (a) How many different ways are there for the kids to line up?

   (b) Mildred and Agnes are best friends and both are students in the class. How many different ways are there for the kids to line up so that Mildred and Agnes are next to each other in line?