Name:____

Test I

ICS 6D Spring 2015 Wed, Apr 29, 2015

Instructor: Sandy Irani

Wait until instructed to turn over the cover page. Complete all of the following questions. There are a total of 50 points.

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1.	(0	pomis	, Supp	ose mai	y ou mas	prove	tile 10	110 11115	ruct ut		ong induction:

For all
$$n \ge 4$$
, $P(n)$ is true.

The base case in your proof shows that P(4), P(5), P(6), and P(7) are all true. Fill in the blanks to indicate what must be proven in the inductive step:

For any k such that _____

then _____

2. (3 points) Express the following sum using summation notation:

$$2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9$$
.

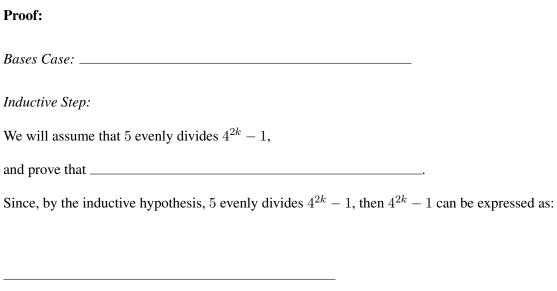
3. (3 points) Give an equivalent expression to the summation below with the last term of the sum outside the summation:

$$\sum_{j=2}^{k+2} (j \cdot 2^j).$$

- 4. (5 points) Below is a recursive deinition for a set S of strings over the alphabet $\{a,b\}$. The set S does not include all strings over the alphabet $\{a,b\}$.
 - Basis: $\lambda \in S$ and $a \in S$.
 - Recursive rules: If x is a string and $x \in S$, then
 - $-xb \in S$
 - $-xba \in S$
 - The only strings that are in S are the ones that can be constructed by starting with one of the strings in the base case and repeatedly applying the recursive rules above.

List all the strings of length 3 in S.

5. (12 points) Fill in the missing statements of the inductive proof below. You can add as many lines you need. Make sure and label where you use the inductive hypothesis in your argument.	as
Theorem 1. For any integer $n \ge 1$, 5 evenly divides $4^{2n} - 1$.	



Then:

$$4^{2(k+1)} - 1 =$$
 $=$

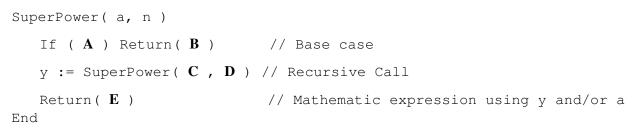
Since $4^{2(k+1)} - 1$ is an integer multiple of 5, then 5 evenly divides $4^{2(k+1)} - 1$.

Recursive Algorithm

The function receives two inputs: a and n. a is a real number and n is an integer such that $n \ge 0$. It should return

$$SuperPower(a, n) = a^{3n+1}$$

Note that in the expression above, the exponent of a is 3n + 1. Below is a recursive algorithm to compute SuperPower(a, n) with some lines missing.



(2 points each question:)

6. For the recursive algorithm, what expression should go in the space labeled A?

(a) a = 1

(c) n = 1

(b) a = 0

(d) n = 0

7. For the recursive algorithm, what expression should go in the space labeled B?

(a) a

(c) 1

(b) a^{3}

(d) 0

8. For the recursive algorithm, what expression should go in the space labeled C?

(a) a DIV 2

(c) n-1

(b) a

(d) a - 1

9. For the recursive algorithm, what expression should go in the space labeled D?

(a) n

(c) a - 1

(b) n-1

(d) n DIV 2

10. For the recursive algorithm, what expression should go in the space labeled E?

(a) a^{3}

(c) y^3

(b) $y \cdot a^3$

(d) $y^3 \cdot a$

- 11. (8 points) Solve the following recurrence relation:
 - $f_0 = 7$
 - $f_1 = 1$
 - $f_n = 2 \cdot f_{n-1} + 3 \cdot f_{n-2}$

- 12. (3 points) The sequence $\{g_n\}$ is defined recursively as follows:
 - $g_0 = 5$
 - $\bullet \ g_n = 2 \cdot g_{n-1} + n$

Theorem 2. For any non-negative integer n, $g_n = 7 \cdot 2^n - n - 2$.

If the theorem above is proven by induction, what must be established in the inductive step?

- (a) For $k \ge 0$, if $g_k = 2 \cdot g_{k-1} + k$, then $g_{k+1} = 7 \cdot 2^{k+1} (k+1) 2$.
- (b) For $k \ge 0$, if $g_k = 7 \cdot 2^k k 2$, then $g_{k+1} = 7 \cdot 2^{k+1} (k+1) 2$.
- (c) For $k \ge 0$, if $g_k = 7 \cdot 2^k k 2$, then $g_{k+1} = 2 \cdot g_k + (k+1)$.
- (d) For $k \ge 0$, if $g_k = 2 \cdot g_{k-1} + k$, then $g_{k+1} = 2 \cdot g_k + (k+1)$.