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## Test I

ICS 6D

Spring 2015

Wed, Apr 29, 2015

*Instructor: Sandy Irani*

Wait until instructed to turn over the cover page. Complete all of the following questions. There are a total of 50 points.

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1. (6 points) Suppose that you must prove the following fact using **strong** induction:

For all  $n \geq 4$ ,  $P(n)$  is true.

The base case in your proof shows that  $P(4)$ ,  $P(5)$ ,  $P(6)$ , and  $P(7)$  are all true. Fill in the blanks to indicate what must be proven in the inductive step:

For any  $k$  such that \_\_\_\_\_,

if \_\_\_\_\_,

then \_\_\_\_\_.

2. (3 points) Express the following sum using summation notation:

$$2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9.$$

3. (3 points) Give an equivalent expression to the summation below with the last term of the sum outside the summation:

$$\sum_{j=2}^{k+2} (j \cdot 2^j).$$

4. (5 points) Below is a recursive definition for a set  $S$  of strings over the alphabet  $\{a, b\}$ . The set  $S$  does not include all strings over the alphabet  $\{a, b\}$ .

- Basis:  $\lambda \in S$  and  $a \in S$ .
- Recursive rules: If  $x$  is a string and  $x \in S$ , then
  - $xb \in S$
  - $xba \in S$
- The only strings that are in  $S$  are the ones that can be constructed by starting with one of the strings in the base case and repeatedly applying the recursive rules above.

List all the strings of length 3 in  $S$ .

5. (12 points) Fill in the missing statements of the inductive proof below. You can add as many lines as you need. Make sure and label where you use the inductive hypothesis in your argument.

**Theorem 1.** *For any integer  $n \geq 1$ , 5 evenly divides  $4^{2n} - 1$ .*

**Proof:**

*Bases Case:* \_\_\_\_\_

*Inductive Step:*

We will assume that 5 evenly divides  $4^{2k} - 1$ ,

and prove that \_\_\_\_\_.

Since, by the inductive hypothesis, 5 evenly divides  $4^{2k} - 1$ , then  $4^{2k} - 1$  can be expressed as:

Then:

$$4^{2(k+1)} - 1 = \underline{\hspace{10cm}}$$

$$= \underline{\hspace{10cm}}$$

$$= 5 \cdot (\underline{\hspace{10cm}})$$

Since  $4^{2(k+1)} - 1$  is an integer multiple of 5, then 5 evenly divides  $4^{2(k+1)} - 1$ .

### Recursive Algorithm

The function receives two inputs:  $a$  and  $n$ .  $a$  is a real number and  $n$  is an integer such that  $n \geq 0$ . It should return

$$\text{SuperPower}(a, n) = a^{3n+1}$$

**Note that in the expression above, the exponent of  $a$  is  $3n + 1$ .** Below is a recursive algorithm to compute  $\text{SuperPower}(a, n)$  with some lines missing.

```
SuperPower( a, n )  
    If ( A ) Return( B )           // Base case  
    y := SuperPower( C , D ) // Recursive Call  
    Return( E )                     // Mathematic expression using y and/or a  
End
```

(2 points each question:)

6. For the recursive algorithm, what expression should go in the space labeled A?

- |             |             |
|-------------|-------------|
| (a) $a = 1$ | (c) $n = 1$ |
| (b) $a = 0$ | (d) $n = 0$ |

7. For the recursive algorithm, what expression should go in the space labeled B?

- |           |         |
|-----------|---------|
| (a) $a$   | (c) $1$ |
| (b) $a^3$ | (d) $0$ |

8. For the recursive algorithm, what expression should go in the space labeled C?

- |                        |             |
|------------------------|-------------|
| (a) $a \text{ DIV } 2$ | (c) $n - 1$ |
| (b) $a$                | (d) $a - 1$ |

9. For the recursive algorithm, what expression should go in the space labeled D?

- |             |                        |
|-------------|------------------------|
| (a) $n$     | (c) $a - 1$            |
| (b) $n - 1$ | (d) $n \text{ DIV } 2$ |

10. For the recursive algorithm, what expression should go in the space labeled E?

- |                   |                   |
|-------------------|-------------------|
| (a) $a^3$         | (c) $y^3$         |
| (b) $y \cdot a^3$ | (d) $y^3 \cdot a$ |

11. (8 points) Solve the following recurrence relation:

- $f_0 = 7$
- $f_1 = 1$
- $f_n = 2 \cdot f_{n-1} + 3 \cdot f_{n-2}$

12. (3 points) The sequence  $\{g_n\}$  is defined recursively as follows:

- $g_0 = 5$
- $g_n = 2 \cdot g_{n-1} + n$

**Theorem 2.** For any non-negative integer  $n$ ,  $g_n = 7 \cdot 2^n - n - 2$ .

If the theorem above is proven by induction, what must be established in the inductive step?

- |   |   |
|---|---|
| (a) For $k \geq 0$ , if $g_k = 2 \cdot g_{k-1} + k$ ,<br>then $g_{k+1} = 7 \cdot 2^{k+1} - (k+1) - 2$ . | (c) For $k \geq 0$ , if $g_k = 7 \cdot 2^k - k - 2$ ,<br>then $g_{k+1} = 2 \cdot g_k + (k+1)$ . |
| (b) For $k \geq 0$ , if $g_k = 7 \cdot 2^k - k - 2$ ,<br>then $g_{k+1} = 7 \cdot 2^{k+1} - (k+1) - 2$ . | (d) For $k \geq 0$ , if $g_k = 2 \cdot g_{k-1} + k$ ,<br>then $g_{k+1} = 2 \cdot g_k + (k+1)$ . |