Version A

Quiz 2

ICS 6D
Fall 2014
Monday, Oct 27, 2014
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Instructions:

1. Wait until instructed to turn over the cover page.
2. Write you name on you Scantron form.
3. Write you student ID number in the box labeled ”Subject” on your scantron form.
4. Write the version of your test (A, B, C, or D) in the box labeled ”TEST NO” on your scantron form.

Recursive Algorithm

The function receives two inputs: a and n. a is a real number and n is a non-negative integer. It should return

$$SuperPower(a, n) = a^{3^n}$$

Note that in the expression above, the exponent of a is 3^n. Below is a recursive algorithm to compute SuperPower(a, n) with some lines missing.

```
SuperPower( a, n )
   If ( A ) Return( B )        // Base case
   y := SuperPower( C , D ) // Recursive Call
   Return( E )                 // Mathematic expression using y and/or a
End
```

Inductive Proof

The sequence $\{g_n\}$ is defined recursively as follows:

- $g_0 = 3$
- $g_n = 2 \cdot g_{n-1} + n$

Theorem 1. *For any non-negative integer n, $g_n = 5 \cdot 2^n - n - 2$.*

Below is an inductive proof of the theorem with some lines missing:

Proof:

Bases Case: _____(F)_____

Inductive Step:

Assume _____(G)_____,

and prove: $g_n = 5 \cdot 2^n - n - 2$.

$$\begin{aligned} g_n &= \text{\_\_\_\_\_}(H)\text{\_\_\_\_\_} && \text{(by definition)} \\ &= \text{\_\_\_\_\_}(I)\text{\_\_\_\_\_} && \text{(by the inductive hypothesis)} \\ &= 5 \cdot 2 \cdot 2^{n-1} - 2 \cdot (n-1) - 2 \cdot 2 + n \\ &= 5 \cdot 2^n - 2n + 2 - 4 + n \\ &= 5 \cdot 2^n - n - 2 \end{aligned}$$

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1. For the recursive algorithm, what expression should go in the space labeled A?

   (a) $n = 1$
   (b) $a = 0$
   (c) $a = 1$
   (d) $n = 0$

2. For the recursive algorithm, what expression should go in the space labeled B?

   (a) 0
   (b) a
   (c) 1
   (d) a^3

3. For the recursive algorithm, what expression should go in the space labeled C?

   (a) $a - 1$
   (b) a
   (c) $\lfloor a/2 \rfloor$
   (d) $n - 1$

4. For the recursive algorithm, what expression should go in the space labeled D?

   (a) $n - 1$
   (b) n
   (c) $\lfloor n/2 \rfloor$
   (d) $a - 1$

5. For the recursive algorithm, what expression should go in the space labeled E?

   (a) a^3
   (b) $y^3 \cdot a$
   (c) $y \cdot a^3$
   (d) y^3

6. According to the definition of the sequence $\{g_n\}$ defined under "Inductive Proof". What is g_3?

   (a) 16
   (b) 32
   (c) 35
   (d) 24

7. For the inductive proof, what expression should go in the space labeled F?

   (a) $g_{n-1} = 5 \cdot 2^{n-1} - (n-1) - 2.$
   (b) $g_1 = 2 \cdot g_0 + 1.$
   (c) $n = 0$: $g_0 = 3 = 5 \cdot 2^0 - 0 - 2.$
   (d) $n = 1$: $g_1 = 7 = 5 \cdot 2^1 - 1 - 2.$

8. For the inductive proof, what expression should go in the space labeled G?

   (a) $g_n = 2 \cdot g_{n-1} + n.$
   (b) $g_0 = 3.$
   (c) $g_{n-1} = 5 \cdot 2^{n-1} - n - 2.$
   (d) $g_{n-1} = 5 \cdot 2^{n-1} - (n-1) - 2.$

9. For the inductive proof, what expression should go in the space labeled H?

   (a) $5 \cdot 2^n - n - 2.$
   (b) $2 \cdot g_{n-1} + n.$
   (c) $5 \cdot 2^{n-1} - (n-1) - 2.$
   (d) $2 \cdot g_{n-1} + (n-1).$

10. For the inductive proof, what expression should go in the space labeled I?

    (a) $2 \cdot (5 \cdot 2^{n-1} - (n-1) - 2) + n.$
    (b) $2 \cdot (5 \cdot 2^n - n - 2) + n.$
    (c) $2 \cdot g_{n-1} + n.$
    (d) $5 \cdot 2^{n-1} - (n-1) - 2.$