ICS 6D	**Due: Friday, May 1, 2015, 2:00PM**

Homework 4

Instructor: Sandy Irani — Sections 8.1, 8.2, 8.3

1. Compute the value of the following expressions:
   (a) $344 \text{ mod } 5$.
   (b) $344 \text{ div } 5$.
   (c) $-344 \text{ mod } 5$.
   (d) $-344 \text{ div } 5$.
   (e) $38^7 \text{ mod } 3$.
   (f) $(72 \cdot (-65) + 211) \text{ mod } 7$.
   (g) $(77 \cdot (-65) + 147) \text{ mod } 7$.
   (h) $(44^{12}) \text{ mod } 6$.
   (i) $(17^{12}) \text{ mod } 3$.

2. Give the multiplication table for $\mathbb{Z}_7$.

3. Some numbers and their prime factorizations are given below.

$$\begin{aligned} 140 &= 2^2 \cdot 5 \cdot 7 \\ 1078 &= 2 \cdot 7^2 \cdot 11 \\ 175 &= 5^2 \cdot 7 \\ 25480 &= 2^3 \cdot 5 \cdot 7^2 \cdot 13 \end{aligned}$$

   Express each of the quantities below as a product of primes:
   (a) $gcd(1078, 140)$
   (b) $gcd(1078, 25480)$
   (c) $lcm(1078, 140)$
   (d) $lcm(175, 25480)$
   (e) $lcm(140, 25480)$
   (f) $175 \cdot 25480$
   (g) $25480/140$

4. Suppose that two positive integers x and y are expressed using a common set of primes as follows:

$$\begin{aligned} x &= p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \cdots p_r^{\alpha_r} \\ y &= p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot p_3^{\beta_3} \cdots p_r^{\beta_r} \end{aligned}$$

   (a) What is the prime factorization for $x \cdot y$?
   (b) If $y|x$, give the prime factorization for x/y. (It's OK to have some p^0's in your prime factorization).

5. Let a, b and c be positive integers. Consider the statement:

   If $a|b$ or $a|c$, then $a|bc$.

   (a) Show that the statement is true.

   (b) Show that the converse of the statement is not true by giving a counter-example. (The converse of the statement says that if $a|bc$, then $a|b$ or $a|c$.)

   (c) Is the converse of the statement true if a is a prime number?

6. Group the following numbers according to equivalence mod 13. That is, put two numbers in the same group if they are equivalent mod 13.

$$\{-63, -54, -41, 11, 13, 76, 80, 130, 132, 137\}$$

7. Use the prime number theorem to give an approximation for the number of prime numbers in the range 2 throught $10,000,000$.

8. Compute $gcd(72, 42)$ and write it in the form $72 \cdot s + 42 \cdot t$ for integers s and t.

9. Compute $gcd(80, 61)$ and write it in the form $80 \cdot s + 61 \cdot t$ for integers s and t.

10. Compute $gcd(630, 147)$ and write it in the form $630 \cdot s + 147 \cdot t$ for integers s and t.

11. Find the multiplicative inverse of 52 mod 77. Note that your answer should be a number y in the range from 0 through 76 such that $x \cdot y$ mod $77 = 1$.