ICS 6D **Due: Friday, April 10, 2015, 2:00PM**

Homework 1

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1. The propositional variables p, q, and r have the following truth values:

   - $p = T$
   - $q = F$
   - $r = F$

   What is the truth value of the following compound propositions:

   (a) $(p \wedge q) \rightarrow \neg r$
   (b) $\neg(r \vee \neg p \vee \neg q)$
   (c) $\neg(q \wedge r) \rightarrow \neg p$
   (d) $\neg(q \wedge r) \vee (p \wedge \neg r)$

2. Which statements below evaluate to true?

   (a) If 3 is a prime number then 4 is also a prime number.
   (b) If January has 28 days, then 4 is a prime number.

3. Show that the expressions $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent by using a truth table.

4. Define the following propositions:

   - s: a person is a senior.
   - v: a person is at least seventeen years of age.
   - p: a person is allowed to park in the parking lot.

   Express each of the following English sentences with a logical expression:

   (a) If a person is allowed to park in the parking lot then they are a senior and at least seventeen years of age.
   (b) A person can park in the parking lot if they are a senior or at least seventeen years of age.

5. In this problem, the domain of discourse is the set of all integers. Which statements are true? If an existential statement is true, give an example. If a universal statement is false, give a counter-example.

   (a) $\exists x(x + x = 1)$
   (b) $\forall x(x^2 - x \neq 1)$
   (c) $\forall x(x^2 - x \neq 0)$
   (d) $\exists x(x + 2 = 1)$
   (e) $\forall x((x < 0 \wedge x > 0) \rightarrow x = 0)$

6. Define the following sets:

   - $A = \{x \in \mathbb{Z} : x \text{ is even}\}$
   - $B = \{x \in R : x \geq 1\}$
   - $C = \{-3, 1, 2, 6, 7, 9\}$
   - $D = \{2, 3, 5, 9, 10, 17\}$

   Indicate whether the following statements are true or false:

   (a) $\pi \in B$
   (b) $A \subseteq B$
   (c) $C \subseteq B$
   (d) $8 \in A \cap B$
   (e) $A \cap C \subseteq B$
   (f) $C \subseteq A \cup B$
   (g) $A \cap C \cap D = \emptyset$
   (h) $|C| = |D|$
   (i) $|C \cap D| = 3$

7. Define the sets $X = \{a, b, c\}$ and $Y = \{1, 2\}$. Show the set $Y \times X$ by listing the elements with set notation.

8. Show each of the sets by listing the elements with set notation. You can specify each element as a string instead of including the parentheses and commas.

   (a) $\{0, 1\}^3$
   (b) $\{0\} \times \{0, 1\}^2 \times \{1\}$
   (c) $\{a, b\} \times \{1, 2\}$

9. What is $|\{0, 1\}^2|$? What is $|\{0, 1\}^3|$?

10. Let $S = \{x, y, z\}$. Give $P(S)$, the power set of S. (Use set notation and list the elements).

11. Let $S = \{x, y, z\}$. Which of the following statements are true?

    (a) $x \in P(S)$
    (b) $\{x\} \in P(S)$
    (c) $\emptyset \in P(S)$
    (d) $\emptyset \in S$.
    (e) $\{x, y\} \in S$.

12. Use set notation to denote each of the sets below. Do not use ellipses (...) in your expression. You may need to use set operations in your expression.

    (a) The set of all binary strings whose length is in the range from 6 through 8 (inclusive).
    (b) The set of all binary strings of length 5 that start with a 0.
    (c) The set of all strings (of any length) that are all 1's.

13. For each of the functions below, indicate whether the function is onto, one-to-one or both. If the function is not one-to-one, give an example showing why. If the function is not onto, give an element y of the target set for which there is no element in the domain that maps on to y.

    (a) $f : R \to R$. $f(x) = x^2$.

    (b) $g : R \to R$. $g(x) = x^3$.

    (c) $h : Z \to Z$. $g(x) = x^3$.

14. For each of the functions below, indicate whether the function is onto, one-to-one or both. If the function is not onto, specify its range by listing the elements using set notation. If the function is not one-to-one, give an example showing why. If the function is a bijection, give its inverse.

    (a) $f : \{0,1\}^3 \to \{0,1\}^4$. The output of f is obtained by copying the first bit and adding it to the end of the string. For example, $f(011) = 0110$ and $f(101) = 1011$.

    (b) $g : \{0,1\}^3 \to \{0,1\}^3$. The output of g is obtained by first dropping the last bit of then string and the copying the first bit and adding it to the end of the string. For example $g(010) = 010$ and $g(110) = 111$.

    (c) $h : \{0,1\}^3 \to \{0,1\}^3$. The output of h is obtained by reversing the bits. For example $h(001) = 100$ and $h(110) = 011$.

    (d) Let P_7 be the set of all 7-bit strings that are palindromes. (A palindrome is a string that is the same read forwards and backwards. For example, 0110110 is an element of P_7). $d : P_7 \to \{0,1\}^4$. The function d drops the last three bits of the string. For example, $d(0110110) = 0110$.

15. Let f be a function $f : \{0,1\}^2 \to \{0,1\}^3$. Is it possible that f is a bijection? Why or why not?