Homework 4

Due: May 29, 2013

1. Consider the problem whose input is a graph with integer weights and asks whether the minimum length tour is unique. For what class in the polynomial hierarchy is this problem complete? Prove your answer.

2. Show that if $\mathbf{NP} \subseteq \mathbf{TIME}(n^{\log n})$, then $\mathbf{PH} \subseteq \cup_{k \geq 1} \mathbf{TIME}(n^{\log^k n})$.

3. **FP** is the set of functions from $\{0,1\}^*$ to $\{0,1\}^*$ that can be computed by a deterministic Turing Machine in polynomial time. Show that computing the permanent of a matrix with integer entries can be done in $\mathbf{FP}^{\#\mathbf{SAT}}$. Note that the integer entries may be negative but you will get partial credit if you prove this under the restricion of non-negative entries.

4. Define a language L to be *downward self-reducible* if there's a polynomial-time algorithm R that for any n and $x \in \{0,1\}^n$, $R^{L_{n-1}}(x) = L(x)$ where by L_k we denote an oracle that solves L on inputs of size at most k. Prove that if L is downward-self-reducible, then $L \in$ PSPACE.

5. Recall the definition of QSAT:

$$\text{QSAT } = \{\Phi(x_1, \ldots, x_n) \mid \exists x_1 \forall x_2 \ldots \forall x_n \Phi(x_1, \ldots, x_n) = 1\},$$

   where Φ is a 3-CNF formula. Show that $P^{QSAT} = NP^{QSAT}$.