Homework 2

Due: April 29, 2013

1. Prove that the problem of deciding whether a digraph is strongly connected is complete for **NL**.

2. Define a *coding* κ to be a mapping from Σ to Σ. Note that κ need not be one-to-one. If $x = \sigma_1 \dots \sigma_n$, where each $\sigma_i \in \Sigma$, then we define $\kappa(x) = \kappa(\sigma_1) \dots \kappa(\sigma_n)$. If L is a language, then $\kappa(L)$ is defined to be $\{\kappa(x) | x \in L\}$.

   (a) Prove that NP is closed under codings. That is, show that if $L \in$ NP and κ is a coding defined on the alphabet of L, then $\kappa(L) \in$ NP.

   (b) We expect that P is not closed under codings, but we can not prove this without establishing that P $\neq$ NP. Instead, show that P is closed under codings if and only if P = NP.

3. Prove that if *every* unary language in **NP** is also in **P**, then **EXP**= **NEXP**. Recall that a language is unary if and only if it is a subset of 1^*.

4. Say that class C_1 is *superior* to C_2 if there is a language L_1 in C_1 such that for every language $L_2 \in C_2$ and every n sufficiently large, there is an input whose length is between n and n^2 on which L_1 and L_2 differ. That is, there is a string x whose length is between n and n^2 such that either $x \in L_1$ and $x \notin L_2$ or $x \notin L_1$ and $x \in L_2$.

   (a) Is $\mathbf{DTIME(n^4)}$ superior to $\mathbf{DTIME(n)}$?

   (b) Why does our proof of the Non-deterministic Time Hierarchy not prove that $\mathbf{NTIME(n^{1.5})}$ is superior to $\mathbf{NTIME(n)}$?

5. In the Arora-Boak text gives an alternative definition of the class **NL** which makes use of a Turing Machine with a special read-once tape. The head on a read-once tape starts at the left-most end of the non-blank symbols written on the tape and can only move to the right or stay in the same place (i.e. it can never move left). The alternative definition says that a language L is in **NL** if there is a deterministic Turing Machine M (called a *verifier*) with a special read-once tape and a polynomial p such that for every $x \in \Sigma^*$,

$$x \in L \Leftrightarrow \exists u \in \Sigma^{p(|x|)} \text{ such that } M(x, u) = 1,$$

   where $M(x, u)$ is the output of M when x is placed on the input tape and u is placed on its special read-once tape and M uses $O(\log n)$ space on its work tape for every input x.

   Prove that this definition is equivalent to the definition using non-deterministic Turing Machines discussed in class.

6. Prove that if in the above definition, the read-once tape is replaced with a read-only tape (which could be read many times), then the resulting class is **NP**.