

CS 264

Quantum Computing

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Course Overview:

What could we compute if we could build a large-scale quantum computer?

What could we not compute?

First need to describe what a quantum computer is

⇒ Model of computation

Will start with a brief introduction to the postulates of quantum mechanics

Course Overview (model of computation)

Classical Computing: Turing Machine

Precise way to understand resources:

steps = time

tape cells = space.

Extended Church-Turing Thesis

Any physically realizable computer

can be efficiently simulated

by a Turing Machine (or Random Access Machine or cellular automata)

polynomial time overhead.

Course Overview (complexity classes).

Define complexity classes based on model of computation

Sets of problems that can be solved with a particular limit on resources.

Resources scale with the size of the problem

P = Set of problems that can be solved on a classical computer in time at most $p(n)$

fixed polynomial

→ # of bits required to specify the input.

Will define BQP analogously for a quantum computer

Course Overview

Quantum Algorithms:

What problems can be solved more efficiently on a quantum computer than a classical computer?

What problems do we expect that we can't solve efficiently on a quantum computer?

Last 3 weeks: error correction.

Postulates of Quantum Mechanics

Quantum Mechanics is a mathematical framework for describing nature

Not a description for nature on its own.

Means of describing

States of a physical system

How states change over time

What information can be extracted from the system.

Postulates of Quantum Mechanics

Superposition

Quantum Mechanics: a generalization of probability

Classical World: bit is 0 or 1.

Value of bit unknown can be expressed as a probability distribution.

p_0 = prob bit is 0

p_1 = prob bit is 1.

$$p_0, p_1 \geq 0$$

$$p_0 + p_1 = 1$$

$(p_0, p_1) \rightsquigarrow L_1 \text{ norm} = 1.$

Postulates of Quantum Mechanics

Superposition

Quantum bit (qubit)

Simultaneously partially 0 + partially 1

"amplitude" (instead of probability)

α_0 - amplitude for 0

α_1 - amplitude for 1.

$\alpha_0, \alpha_1 \in \mathbb{C}$ (complex numbers)

State: $\alpha_0 |0\rangle + \alpha_1 |1\rangle$

↳ Dirac notation.

Postulates of Quantum Mechanics

What does a negative (or complex) amplitude mean?

If we measure the value of the qubit $\alpha_0|0\rangle + \alpha_1|1\rangle$

With probability $|\alpha_0|^2$

Outcome is $|0\rangle$

New state is $|0\rangle$

With probability $|\alpha_1|^2$

Outcome is $|1\rangle$

New State is $|1\rangle$

Postulates of Quantum Mechanics

What does a negative (or complex) amplitude mean?

If we measure the value of the qubit $\alpha_0|0\rangle + \alpha_1|1\rangle$

With probability $|\alpha_0|^2$

Outcome is $|0\rangle$

New state is $|0\rangle$

Measurement changes the state

Need:

With probability $|\alpha_1|^2$

Outcome is $|1\rangle$

New State is $|1\rangle$

$$|\alpha_0|^2 + |\alpha_1|^2 = 1.$$

L_2 norm of (α_0, α_1) is 1.

Postulates of Quantum Mechanics

What about n qubits?

Classical bits state = $x \in \{0, 1\}^n$

n -bit strings

Classical state with uncertainty

$P_x = \text{prob state} = x$

$(P_0, P_1, \dots, P_{2^n-1})$

L_1 norm
(sum of entries)
is 1.

Quantum State

$\sum_{x \in \{0, 1\}^n} \alpha_x |x\rangle$

$(\alpha_0, \alpha_1, \dots, \alpha_{2^n-1})$

L_2 norm is 1

$$\sum_x |\alpha_x|^2 = 1.$$

Postulates of Quantum Mechanics

Measurement

Example: 3 qubits.

$$\begin{aligned} & \alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle \\ & + \alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle \\ & (\alpha_{000}, \alpha_{001}, \alpha_{010}, \alpha_{011}, \alpha_{100}, \alpha_{101}, \alpha_{110}, \alpha_{111}) \end{aligned}$$

Measure all 3 qubits

Outcome is 010 w/ prob $|\alpha_{010}|^2$

Afterwards state is $|010\rangle$

$$(0, 0, \overset{010}{\uparrow} 1, 0, 0, 0, 0, 0)$$

If we measure qubits again, the outcome is determined.

Postulates of Quantum Mechanics

Measurement

Example: 3 qubits.

$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle \\ + \alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle$$

Measure first qubit only

$$\text{Prob outcome is } 0 = P_0 = |\alpha_{000}|^2 + |\alpha_{001}|^2 + |\alpha_{010}|^2 + |\alpha_{011}|^2$$

Afterwards state collapses to:

$$\frac{\alpha_{000}}{\sqrt{P_0}}|000\rangle + \frac{\alpha_{001}}{\sqrt{P_0}}|001\rangle + \frac{\alpha_{010}}{\sqrt{P_0}}|010\rangle + \frac{\alpha_{011}}{\sqrt{P_0}}|011\rangle$$

$$(\alpha_{000}/\sqrt{P_0}, \alpha_{001}/\sqrt{P_0}, \alpha_{010}/\sqrt{P_0}, \alpha_{011}/\sqrt{P_0}, 0, 0, 0, 0)$$

Postulates of Quantum Mechanics

Evolution

Evolution of a quantum system over time is Unitary

In matrix notation it is a Unitary matrix.

State : $\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix}$ with L_2 norm = 1

U unitary $N \times N$ matrix then

$$[U] \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix} \text{ also has } L_2 \text{ norm} = 1.$$

Continuous Quantum Spaces

So far only discussed finite dimensional Hilbert spaces.

Measurement has a finite set of outcomes.

In physics, often concerned with quantities in a continuous space
(e.g. the location of a particle)

State $\psi(x)$ continuous function of x (or $\vec{x}$)
 $\psi(x)$ is complex

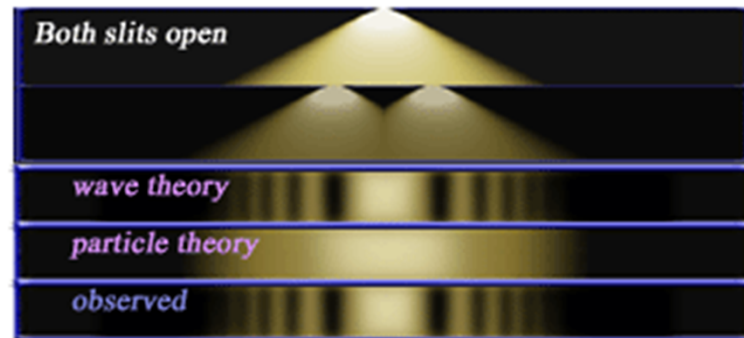
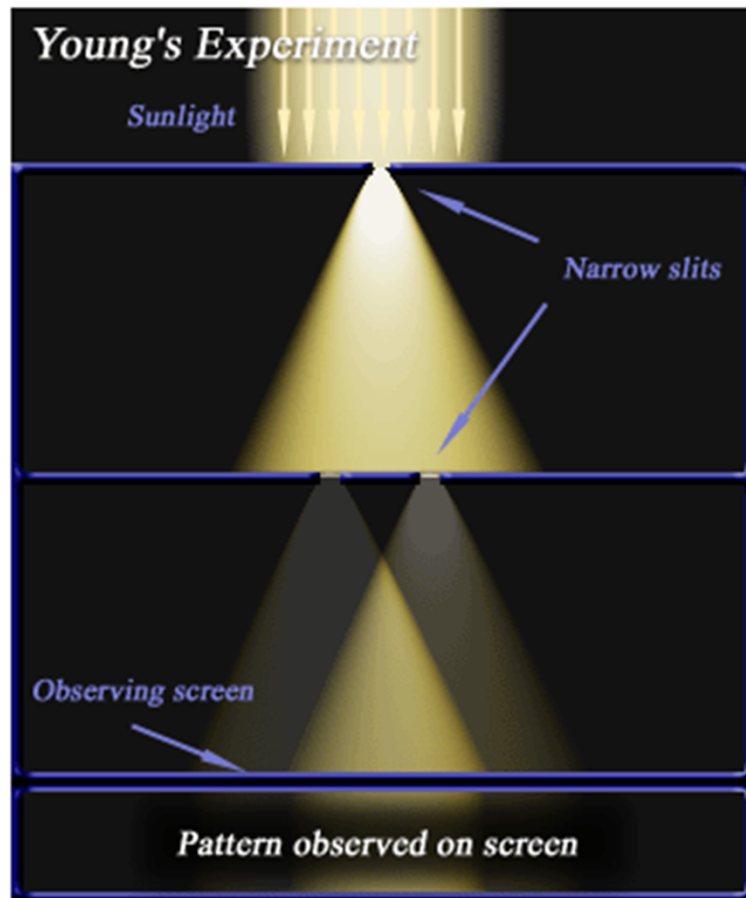
Normalization: $\int_{-\infty}^{\infty} \psi^*(x) \psi(x) dx = 1.$

If we measure location, the prob in range $[a, b]$:

$$\int_a^b \psi^*(x) \psi(x) dx$$

Young's Double Slit Experiment

Negative Interference



Wave patterned formed by one photon
at a time
(i.e. no interference between different photons)

Location where photon hits is probabilistic.

Classical Explanation:

$$P(x) = (\text{prob slit 1})p(x|\text{slit 1}) + (\text{prob slit 2})p(x|\text{slit 2})$$

(weighted average of patterns from single slit cases)

Quantum Explanation

$$\psi(x) = \alpha_1 \psi_1(x) + \alpha_2 \psi_2(x)$$

state from slit 1

state from slit 2

Can have
negative
interference.

Postulates of Quantum Mechanics

Why complex? (why not just negative)

Consider two qubit states:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Continuous transition from $|+\rangle$ to $|-\rangle$

Intermediate state of form:

$$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle) \quad \theta: 0 \rightarrow \pi.$$

Postulates of Quantum Mechanics

Why the L_2 norm?

Why shouldn't the probability of an outcome be amplitude to the 4th power?

$L_1 + L_2$ are only two that allow for non-trivial norm-preserving linear transforms.

Using A Quantum Mechanical System to Compute

- ① Storing information (state vector)
- ② Manipulating Information
(unitary evolution)
- ③ Extracting information
(quantum measurement)

Linear Algebra

Hilbert Space: Complex vector space
with inner product

N distinguishable states: $|1\rangle, |2\rangle, \dots, |N\rangle$
(e.g. $N = 2^n$ n -bit strings)

$|1\rangle \dots |N\rangle$ form an orthonormal basis
of the Hilbert space

$\Rightarrow$ any state can be expressed as a
linear combination of the $|j\rangle$'s.

$$|\psi\rangle = \alpha_1 |1\rangle + \alpha_2 |2\rangle + \dots + \alpha_N |N\rangle$$

Inner product of any two: $(|j\rangle, |k\rangle) = \delta_{jk}$

Properties of the inner product

- Linear in second argument:

$$(|v\rangle, \sum_k \lambda_k |w_k\rangle) = \sum_k \lambda_k (|v\rangle, |w_k\rangle)$$

- $(|v\rangle, |w\rangle) = (|w\rangle, |v\rangle)^*$

- $(|v\rangle, |v\rangle) \geq 0$
Equality only if $|v\rangle = 0$.

Linear Algebra

Once we have specified a basis can express:

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} \quad |\psi'\rangle = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}$$

Inner product defined as:

$$(|\psi\rangle, |\psi'\rangle) = \sum_{j=1}^N \alpha_j^* \beta_j$$

Dirac notation: Dual of $|\psi\rangle$ is $\langle\psi| = (\alpha_1^*, \dots, \alpha_N^*)$

$$(|\psi\rangle, |\psi'\rangle) = \langle\psi|\psi'\rangle = \underbrace{(\alpha_1^*, \dots, \alpha_N^*)}_{1 \times N} \underbrace{\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}}_{N \times 1}$$

L_2 norm = 1:

$$\langle\psi|\psi\rangle = 1$$

(scalar)

$$\Rightarrow 1 \times 1$$

Linear Algebra

1-bit example: States live in a 2-dimensional Hilbert space.

Basis: $|0\rangle, |1\rangle$ ("standard" basis)

Ortho normal: $\langle 0|0\rangle = \langle 1|1\rangle = 1$ $\langle 0|1\rangle = \langle 1|0\rangle = 0$

Any state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

When $|0\rangle + |1\rangle$ are understood: $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

Another ortho normal basis:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Can verify $\langle +|+\rangle = \langle -|-\rangle = 1$ $\langle +|-\rangle = \langle -|+\rangle = 0$

Can express any state as $|\phi\rangle = \alpha|+\rangle + \beta|-\rangle$

Linear Algebra

Hilbert space V , dimension N

Once a basis is fixed can express linear operator as an $N \times N$ matrix.

$$A|v\rangle = |v'\rangle$$

Adjoint of A is the unique linear operator that satisfies:

$$(|v\rangle, A|w\rangle) = (A^\dagger|v\rangle, |w\rangle)$$

$$\begin{array}{ccc} \parallel & \Downarrow & \\ \langle v|A|w\rangle & \boxed{} & |w\rangle \end{array}$$

$$\text{dual of } A^\dagger|v\rangle = \langle v|A$$

If $A + A^\dagger$ are expressed in same basis then
 $A^\dagger$ conjugate-transpose of A

transpose matrix;
complex conjugate
of each entry.

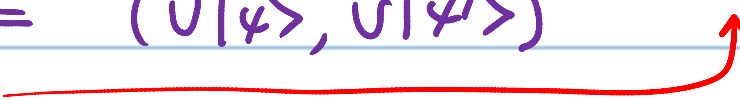
Linear Algebra

Evolution of a quantum system over time can be expressed as a unitary operator U :

Start in $|\psi\rangle$ end up in $U|\psi\rangle$

Following Conditions are Equivalent:

- * U is norm preserving
- * U preserves inner product:
$$(|\psi\rangle, |\psi'\rangle) = (U|\psi\rangle, U|\psi'\rangle)$$
- * $U^\dagger U = U U^\dagger = I$
- * U expressed as matrix, rows form an orthonormal basis.
- * U expressed as a matrix, columns form an orthonormal basis.

$$\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$


Linear Operators

Can use Dirac notation to express operators using "outer bracket" notation.

$$\underbrace{|\psi'\rangle}_{N \times 1} \underbrace{\langle \psi|}_{1 \times N} = \underbrace{\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}}_{N \times N} (\alpha_1^* \dots \alpha_N^*) = \sum_j \left[\begin{array}{c} \vdots \\ \beta_j \alpha_k^* \end{array} \right]$$

$$|\psi'\rangle\langle\psi| \text{ applied to } |\phi\rangle = \langle\psi|\phi\rangle|\psi'\rangle$$

Can build up more complex linear operators by taking linear combinations:

$$\alpha |\psi\rangle\langle\psi| + \beta |\psi\rangle\langle\psi| + \gamma |\phi\rangle\langle\psi| \dots$$

Express A in Outer Bracket Notation

Start with identity.

Ortho
normal Basis $|\phi_1\rangle \dots |\phi_N\rangle$

Any state $|v\rangle = \alpha_1 |\phi_1\rangle \dots \alpha_N |\phi_N\rangle$

Note $\langle \phi_i | v \rangle = \alpha_i$

$$\underbrace{\left(\sum_{j=1}^N |\phi_j\rangle \langle \phi_j| \right)}_{\text{Identity}} |v\rangle = \sum_{j=1}^N \langle \phi_j | v \rangle |\phi_j\rangle = \sum_{j=1}^N \alpha_j |\phi_j\rangle = |v\rangle$$

Express A in Outer Bracket Notation

$$A = I A I = \left[\sum_j |\phi_j\rangle \langle \phi_j| \right] A \left[\sum_k |\phi_k\rangle \langle \phi_k| \right]$$

$$= \sum_{j,k} |\phi_j\rangle \underbrace{\langle \phi_j | A | \phi_k \rangle}_{\text{scalar}} \langle \phi_k|$$

$$= \sum_{j,k} \langle \phi_j | A | \phi_k \rangle |\phi_j\rangle \langle \phi_k|$$

Express A in basis $|\phi_1\rangle \dots |\phi_n\rangle$

$$\begin{array}{c}
 (0 \dots 1 \dots 0) \\
 \uparrow \\
 \text{1 in location } j
 \end{array}
 \begin{array}{c}
 \left[\begin{array}{c} A \\ \hline \end{array} \right] \\
 \downarrow j
 \end{array}
 \begin{array}{c}
 \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix} \\
 \uparrow \\
 \text{1 in location } k
 \end{array}
 = A_{jk}$$

$$\begin{array}{c}
 \parallel \\
 \langle \phi_j | A | \phi_k \rangle
 \end{array}$$

Eigenvectors and Eigenvalues

linear operator A , non-zero $|\phi\rangle$

$$A|\phi\rangle = \lambda |\phi\rangle$$

λ eigenvalue $|\phi\rangle$ eigenvector

Diagonal representation of A :

$$\sum \lambda_i |\phi_i\rangle \langle \phi_i| \quad |\phi_i\rangle \text{ are an orthonormal set.}$$

Theorem: A has a diagonal representation if and only if it is normal (i.e. $AA^\dagger = A^\dagger A$)

Linear Algebra Projectors

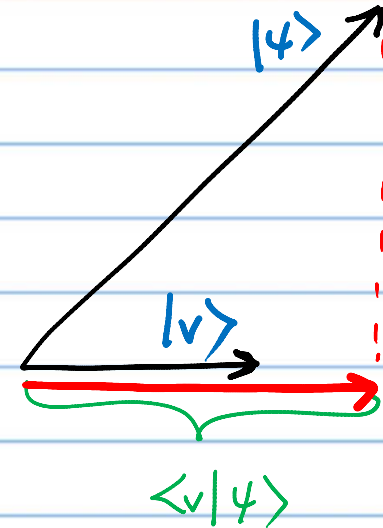
(special class of linear operators)

Unit length

State $|v\rangle$. $P_v = |v\rangle\langle v|$

Projects any state onto $|v\rangle$:

$$P_v |\psi\rangle = \langle v|\psi\rangle |v\rangle$$



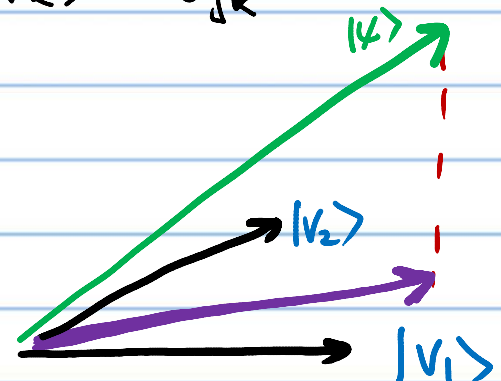
Can also project onto subspaces of higher dimension:

$$|v_1\rangle \dots |v_r\rangle \in \mathbb{C}^N$$

$$\langle v_i | v_k \rangle = \delta_{ik}$$

$$P = \sum_i |v_i\rangle\langle v_i|$$

Note $\cdot P^2 = P$



Generalized Measurements:

Measurement specifies orthonormal basis $\{|v_i\rangle\}$ and real value r_i for each $|v_i\rangle$

If r_i 's are all distinct (non-degenerate)

Measure state $|\phi\rangle$:

Outcome of measurement is r_i w/ prob $|\langle v_i | \phi \rangle|^2$

Afterwards state is $|v_i\rangle$

i.e. $|\phi\rangle = \alpha_1 |v_1\rangle + \alpha_2 |v_2\rangle + \dots + \alpha_N |v_N\rangle$

probability of outcome r_i is $|\alpha_i|^2$

afterwards state is $|v_i\rangle$

Generalized Measurements (non-degenerate)

Our n -qubit example

Measuring all n qubits:

$\{|x\rangle\}$ for all n -bit strings x

r_x = number represented by binary string x .

non-degenerate because each x gives a different r_x

Generalized Measurement

degenerate case
generalization of
non-degenerate case.

r_j 's not distinct

$|v_1\rangle \dots |v_m\rangle$ all have $r_j = R$ $1 \leq j \leq m$

$$P_R = \sum_{j=1}^m |v_j\rangle \langle v_j| \rightarrow \text{projector to subspace spanned by } |v_1\rangle \dots |v_r\rangle$$

Outcome = R with prob $|P|\psi\rangle|^2$

New state: $\frac{P|\psi\rangle}{|P|\psi\rangle|}$

$$|\psi\rangle = \alpha_1 |v_1\rangle + \alpha_2 |v_2\rangle \dots \alpha_m |v_m\rangle + \dots + \alpha_N |v_N\rangle$$

prob outcome = R is $|\alpha_1|^2 + \dots + |\alpha_m|^2$
state becomes $[\alpha_1 |v_1\rangle \dots + \alpha_m |v_m\rangle] / |P|\psi\rangle|$

Generalized Measurements

Our n -qubit example

Measuring first qubit only

$\{ |x\rangle \}$ x : n -bit strings.

r_x = value of the first bit of x .

$$P_0 = \sum_{x: 1^{st} \text{ bit} = 0} |x\rangle\langle x|$$

$$P_1 = \sum_{x: 1^{st} \text{ bit} = 1} |x\rangle\langle x|$$

Measure
 $|\phi\rangle$

Outcome = 0 w/ prob $|P_0|\phi\rangle|^2$

New State is $P_0|\phi\rangle / |P_0|\phi\rangle|$

Generalized Measurement

Measurement Operator: $M = \sum_i r_i |v_i\rangle\langle v_i|$

$|v_i\rangle$'s are eigenvectors of M

r_i 's are eigenvalues: $M |v_i\rangle = r_i |v_i\rangle$

$\langle\phi|M|\phi\rangle$ is the expectation of the outcome of measurement M on $|\phi\rangle$

M is a Hermitian operator:

eigenvalues are real

(mathematically equivalent to $M = M^\dagger$).

Measurement Postulate

Each physically observable quantity
(energy, momentum, location....)
is associated with a Hermitian operator

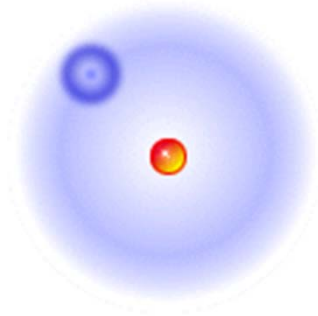
The outcome of measuring such a
quantity is an eigenvalue of the operator

Afterwards the state $|\phi\rangle$ becomes $P|\phi\rangle$

P is the projector onto the subspace
spanned by the eigenvectors whose
eigenvalues are the same as the outcome.

Hamiltonians: an example

Hydrogen Atom



The "state" is the location of the electron:

$$\psi(r, \theta, \phi)$$

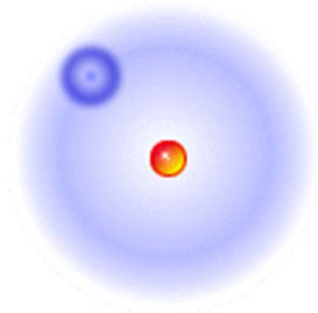
The Hamiltonian describes the energy
As a function of the location of the electron:

$$\hat{H} = -\frac{h^2}{2m_e} \nabla^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

Hamiltonians: an example

Hydrogen Atom



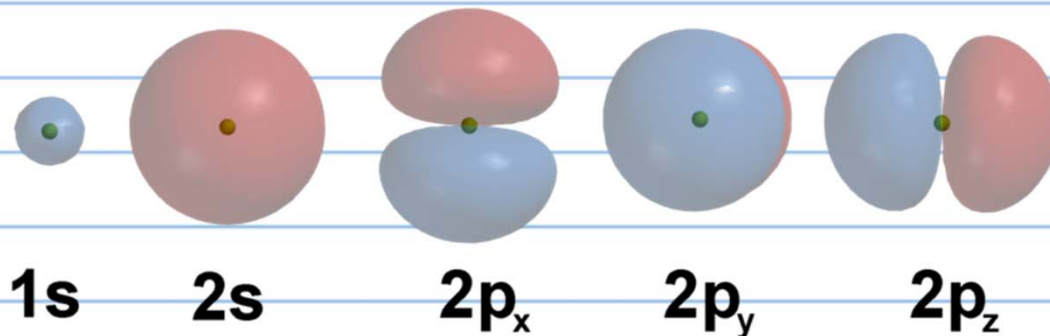
The "state" is the location of the electron:

$$\psi(r, \theta, \phi)$$

The Hamiltonian describes the energy
As a function of the location of the electron:

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$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi^2}$$



Hydrogen Atom, cont.

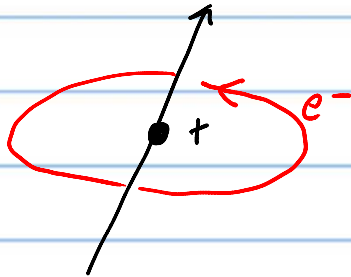
Even systems that appear to be completely continuous (e.g. energy, electron spin...) can give rise to discrete, quantized eigenvalues.

... potential implementation of a qubit.

Stern-Gerlach Experiment

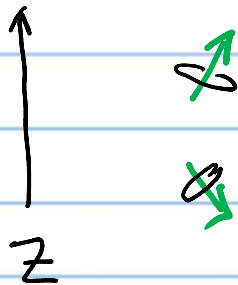
1922 - Silver
1927 - Hydrogen

Atomic
Spin.



Loop of "current" creates
magnetic dipole moment
(bar magnet)

Experimental apparatus: direction of deflection depends on $\hat{z}$ component of the atoms magnetic dipole moment.



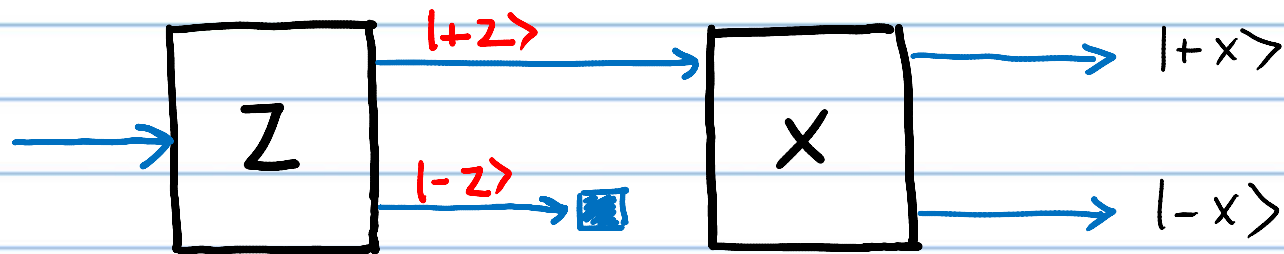
- Atoms deflected in discrete (quantized beams).

- Hydrogen atoms produced two beams. (would expect no moment).

⇒ Electrons have intrinsic spin $|\uparrow\rangle$ $|\downarrow\rangle$

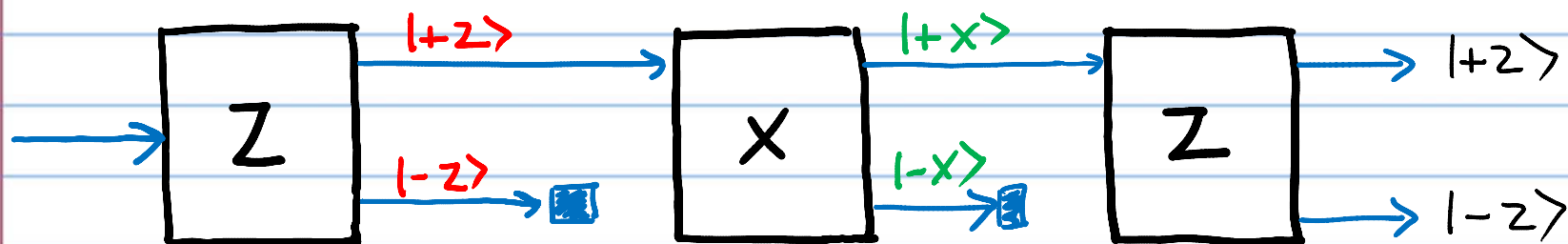
Cascade two deflectors

Second turned on its side - measures spin
in $\hat{X}$ direction.



Would have expected a single beam since
magnetic moment $|+z\rangle$ has no
 X component

Perhaps atoms have magnetic dipole moment along each axis independently...



Quantum Explanation

Define $|+z\rangle = |0\rangle$ $| - z\rangle = |1\rangle$

Z : measures spin in $|0\rangle$ $|1\rangle$ basis

X : measures spin in $|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$
 $|-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$
 basis

Using A Quantum Mechanical System to Compute

① Storing information (state vector)

⇒ ② Manipulating Information
(unitary evolution)

③ Extracting information
(quantum measurement)

Schödinger's Equation

Energy operator plays a special role in QM
⇒ Governs time evolution.

$$i\hbar \frac{d|\psi(t)\rangle}{dt} = H |\psi(t)\rangle$$

↑ H = energy operator
called the Hamiltonian.

Take energy eigenstate: $H|\phi_i\rangle = \lambda_i |\phi_i\rangle$

$$\psi(0) = |\phi_i\rangle$$

$$\frac{d|\psi(0)\rangle}{dt} \propto H|\phi_i\rangle \propto |\phi_i\rangle$$

$$\Rightarrow |\psi(t)\rangle = a(t) |\phi_i\rangle$$

Plug back
into
Schröd Eqn...

$$|\psi(t)\rangle = a(t) |\phi_j\rangle \Rightarrow i\hbar \frac{d|\psi(t)\rangle}{dt} = H |\psi(t)\rangle$$

$$i \hbar \frac{d a(t) |\phi_j\rangle}{dt} = H a(t) |\phi_j\rangle = a(t) \lambda_j |\phi_j\rangle$$

$$i \hbar \frac{d a(t)}{dt} = a(t) \lambda_j \Rightarrow a(t) = e^{-i\lambda_j t}$$

$$|\psi(t)\rangle = e^{-i\lambda_j t} |\phi_j\rangle$$

More generally: $|\psi\rangle = \alpha_1 |\phi_1\rangle + \alpha_2 |\phi_2\rangle + \dots + \alpha_N |\phi_N\rangle$

(express general $|\psi\rangle$ a superposition of energy eigenstates)

$$|\psi(t)\rangle = \alpha_1 e^{-i\lambda_1 t} |\phi_1\rangle + \alpha_2 e^{-i\lambda_2 t} |\phi_2\rangle + \dots + \alpha_N e^{-i\lambda_N t} |\phi_N\rangle$$

Each energy eigenstate precessing at different rate.

To manipulate data stored in
quantum mechanical system

Change Hamiltonian (energy)
in a controlled way for some
specific amount of time

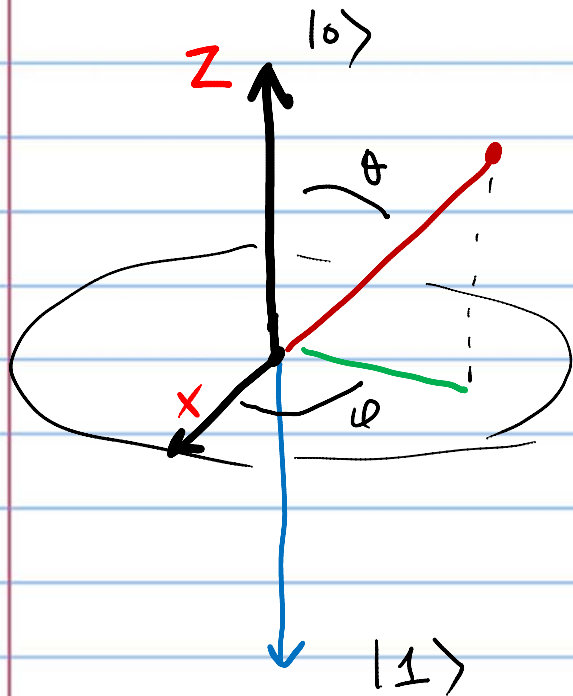
An example,...

Idea: Store qubit in the spin of an electron

Bloch sphere.

represent: $| \psi \rangle = \cos \frac{\theta}{2} | 0 \rangle + e^{i\varphi} \sin \frac{\theta}{2} | 1 \rangle$

note that global phase $e^{i\delta} | \psi \rangle$
cannot be detected by any measurement



- Start with state $|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$

- Apply magnetic field $\vec{B}$

$$\text{new } H = -\vec{\mu} \cdot \vec{B}$$

- Use Schrödinger's Egn to determine how this changes $|\psi\rangle$

($\vec{\mu}$ rotates with axis $\vec{B}$)

- Select $\vec{B}$ and time length t to obtain new state

$$|\psi'\rangle = \cos \frac{\theta'}{2} |0\rangle + e^{i\phi'} \sin \frac{\theta'}{2} |1\rangle$$