Quantum Teleportation

Note Title 4/9/2012

The No-cloning Theorem tells us that there is no Unitary transform that does $|\psi\rangle \otimes |0\rangle \to |\psi\rangle \otimes |\psi\rangle$ for all $|\psi\rangle$.

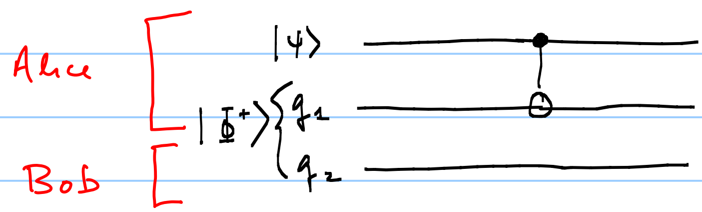
However, if we are willing to destroy the original, we can transmit a qubit using an entangled pair (plus 2 bits of classical communication).

Alice would like to transmit $|\psi\rangle$ (single qubit state) to Bob. Alice and Bob share the entangled state

$$|\phi\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$

Alice has the first qubit and Bob has the second.

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$



Start with $|\psi\rangle \otimes |\Phi^+\rangle = (a_0|0\rangle + a_1|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$

$$= \sum_{\substack{i=0,1\\ j=0,1}} \frac{1}{\sqrt{2}} a_i |i\rangle|jj\rangle$$

After CNOT $\displaystyle = \sum_{\substack{i=0,1\\ j=0,1}} \frac{a_i}{\sqrt{2}} |i, i\oplus j, j\rangle$

Now Alice measures middle qubit.

$l=0$ $(i=j)$ $\displaystyle \sum_{j=0,1} \frac{a_j}{\sqrt{2}}|jjj\rangle \xrightarrow[\text{normalize}]{} a_0|00\rangle + a_1|11\rangle$

Alice

Bob

$l = 1 \quad (i \neq j) \qquad \frac{a_0}{\sqrt{2}}|01\rangle + \frac{a_1}{\sqrt{2}}|10\rangle \xrightarrow{\text{normalize}} a_0|01\rangle + a_1|10\rangle$

Alice sends l to B.

If $l = 1$, Bob will toggle his qubit:

$a_0|00\rangle + a_1|11\rangle$

Almost done, except that Alice's qubit is still entangled with Bob's. If she does a measurement, then it will destroy Bob's copy.

$$\begin{array}{ll} \text{Alice} & \text{———}\boxed{H}\text{—}\boxed{H}\text{—} \\ \text{Bob} & \text{——————————} \end{array}$$

$a_0|+\rangle|0\rangle + a_1|-\rangle|1\rangle$

$= \frac{a_0}{\sqrt{2}}\left(|00\rangle + |10\rangle\right) + \frac{a_1}{\sqrt{2}}\left(|01\rangle - |11\rangle\right)$

Measurement

$\text{Prob} = \frac{|a_0|^2}{2} + \frac{|a_1|^2}{2} = \frac{1}{2}$

$|0\rangle(a_0|0\rangle + a_1|1\rangle)$ $\qquad$ $|1\rangle(a_0|0\rangle - a_1|1\rangle)$

Alice sends measurement result to Bob

Bob does nothing $\qquad$ Bob does phase flip

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$|1\rangle(a_0|0\rangle + a_1|1\rangle)$