EPR Experiment (source: Nielsen + Chuang)

Note Title 4/6/2012

In the classical view of nature, an object has inherent physical properties. Measurements only reveal those properties – they exist independently of observation.
(Example flipping a coin – can theoretically predict how it will fall given enough information: force + trajectory of toss, air resistance, wind, etc.)

In the quantum view of the world objects don't have inherent physical properties until they are measured (at least some properties are not determined until measurement).

Many physicists initially rejected this view of nature.
Einstein, Podolsky, Rosen (EPR) proposed a thought experiment to demonstrate that Quantum Mechanics is not a complete view of nature.
The idea is that there is some element of reality not present in Quantum Mechanics.

Consider an entangled pair $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
Alice and Bob each hold a qubit and are separated by some distance. If Alice measures her qubit then she can predict Bob's qubit perfectly. Alice has this information instantly – faster than the information could travel from Bob.
EPR assertion: since Alice can predict Bob's bit it should be included in reality (encoded somewhere in nature) before Alice's measurement.

This is the same reasoning that says viewing a coin flip as random is not a complete view of nature.

Perhaps the entangled qubits decided together beforehand what the outcome of the measurement would be. (we just didn't know it until we measure).

This assertion was experimentally invalidated with a result known as Bell's Inequality. The idea is to perform a thought experiment & analyze the situation using our classical understanding of nature and then using quantum mechanics. Each approach will lead to a different outcome. This can then be validated experimentally.

Classical version: Alice and Bob each share a string S of arbitrary length. This string corresponds to the "hidden" information stored in two entangled qubits. Then they each receive two independent random bits: X_A to Alice and X_B to Bob. Their goal is to output two quibits: a from Alice and b from B such that $X_A \wedge X_B = \underbrace{a \oplus b}_{a+b \bmod 2}$.

It can be shown that the optimal strategy for Alice + Bob is to output $a = b = 0$ in which case their probability of success is $.75$

In the Quantum Mechanical version of this protocol Alice and Bob share an entangled pair of qubits $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Alice has one qubit and Bob has the other.

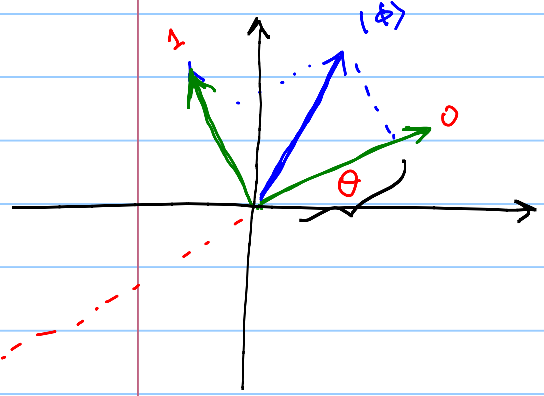
If the strange behavior of this entangled state can be explained by a hidden variable theory then every quantum protocol has a classical counterpart.

Here's a quantum protocol that does strictly better than any classical protocol:

if $X_A = 0$ then Alice measures in $-\pi/16$ basis
if $X_A = 1$ then Alice measures in $3\pi/16$ basis
if $X_B = 0$ then Bob measures in $\pi/16$ basis
if $X_B = 1$ then Bob measures in $-3\pi/16$ basis.

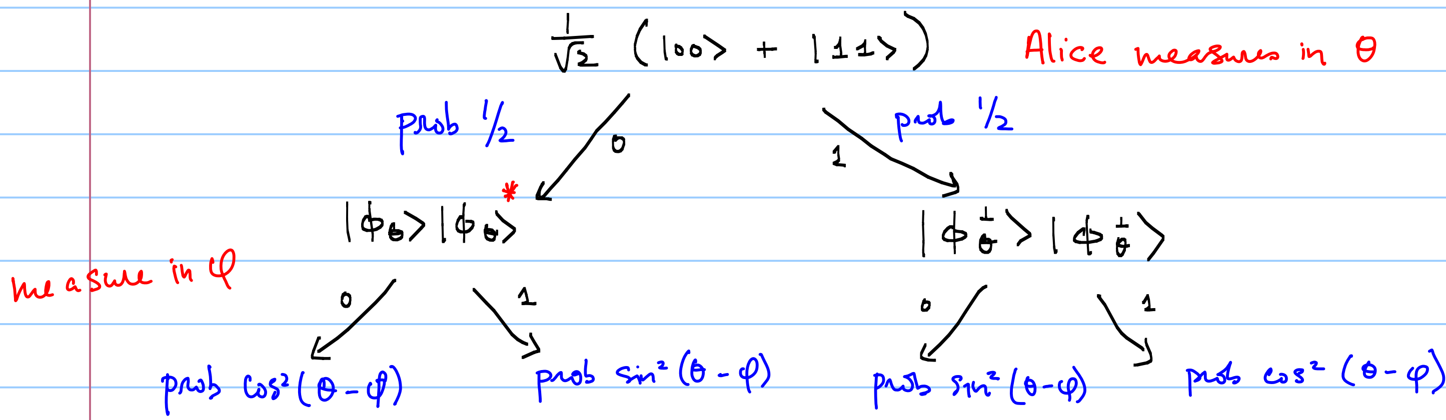
What does it mean to measure in θ basis?

$0 - |\phi_\theta\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$
$1 - |\phi_\theta^\perp\rangle = \sin\theta\,|0\rangle - \cos\theta\,|1\rangle$.



Same as rotate by $-\theta$ and measure in $|0\rangle, |1\rangle$ basis.

If Alice measures in θ basis + Bob measures in φ basis: probability of same outcome in $\cos^2(\theta - \varphi)$ + different outcome is $\sin^2(\theta - \varphi)$.



* Note that $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) =$

$$\tfrac{1}{\sqrt{2}}\Big(\big(\cos\theta|\phi_\theta\rangle + \sin\theta|\phi_\theta^\perp\rangle\big)|0\rangle + \big(\sin\theta|\phi_\theta\rangle - \cos\theta|\phi_\theta^\perp\rangle\big)|1\rangle \Big)$$

$$= \tfrac{1}{\sqrt{2}}\big(|\phi_\theta\rangle|\phi_\theta\rangle + |\phi_\theta^\perp\rangle|\phi_\theta^\perp\rangle \big)$$

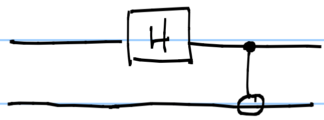
Probability of success:

$X_A = X_B = 0$ want outcome of measurement to be the same. This happens w/ prob $\cos^2(\pi/8) = \sim .853\ldots$

Similarly for $X_A = 1, X_B = 0$ and $X_A = 0\ X_B = 1$.

If $X_A = X_B = 1$ want outcomes to be different. This happens with probability $\sin^2(3\pi/8)$

Bell States: Consider the following circuit:



it yields the four following states on inputs $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = |\Phi^+\rangle$$
$$|01\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) = |\Psi^+\rangle$$
$$|10\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right) = |\Phi^-\rangle$$
$$|11\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right) = |\Psi^-\rangle$$

These are known as the Bell states. They form an orthonormal basis over 2-qubit states. They are all maximally entangled.

Above we argued that $|\Phi^+\rangle$ can be expressed as $\frac{1}{\sqrt{2}}\left(|\phi_\theta\rangle|\phi_\theta\rangle + |\phi_\theta^\perp\rangle|\phi_\theta^\perp\rangle\right)$. We used the fact that the $|\phi_\theta\rangle$ form a special class of states whose amplitude is real.

However the state $|\Psi^-\rangle$ is truly invariant under change of basis. Consider an arbitrary 1-qubit state $|\phi\rangle$ and let $|\phi^\perp\rangle$ be normalized + orthogonal to $|\phi\rangle$, so $\{|\phi\rangle, |\phi^\perp\rangle\}$ form a basis over all 1-qubit states. Then

$$|\Psi^-\rangle = \pm\left(|\phi\rangle|\phi^\perp\rangle - |\phi^\perp\rangle|\phi\rangle\right)$$