CS 264: Quantum Computation
Homework 5
Spring 2012

Due: June 15, 2012

1. Give the check matrix for the 5-qubit code in standard form. Use this to find $\bar{Z}$ and $\bar{X}$.

2. When a stabilizer S has n generators on n qubits, the subspace V_S is just a single state, called a *stabilizer* state. Show that the four Bell states $|00\rangle \pm |11\rangle$, $|01\rangle \pm |10\rangle$ are stabilizer states and give their stabilizers. Show that the state $|010\rangle - |101\rangle$ is a stabilizer state and give its stabilizer. (In all cases, it is enough to specify a set of generators for the stabilizer.)

3. In this problem, we will construct a 4-qubit code which can detect (but not correct) an error on a single qubit. That is, we will be able to determine whether an error has occurred, but we will not be able to determine which error occurred.

   (a) Show that the three operators in G_4 satisy the conditions for a stabilizer: $g_1 = XXII$, $g_2 = IIXX$, $g_3 = ZZZZ$.

   (b) Find an operator in G_4 which is independent from and commutes with $\{g_1, g_2, g_3\}$.Call this operator $\bar{Z}$.

   (c) Construct a projector which projects onto the space stabilized by the $\langle g_1, g_2, g_3, \bar{Z}\rangle$. Do the same for $\langle g_1, g_2, g_3, -\bar{Z}\rangle$. Your projecter can express your projector as a sum of products of the g_i's and $\bar{Z}$.

   (d) Use the projectors you have constructed to give a basis for V_S.

   (e) Prove that any element of G_4 of weight 1 anti-commutes with at least one of the g_i's.

   (f) Explain how you can use the fact from the previous part to detect whether a 1-qubit error has occurred.