# CS 264: Quantum Computation
# Homework 4
# Spring 2012

Due: June 4, 2012

1. In class we discussed a method to simulate the dynamics of a quantum system when the Hamiltonian is the sum of local terms. We can expand the set of quantum systems that can be simulated to certain classes of non-local Hamiltonians. Give an algorithm to efficiently simulate the Hamiltonian

   $$H = \bigotimes_{j=1}^{n} \sigma_{s_j}^{(j)}.$$

   The superscript indicates the qubit on which the operator is acting. Also, $s_j \in \{0, 1, 2, 3\}$ and $\sigma_0, \sigma_1, \sigma_2, \sigma_3$ represent the Pauli operators: $\sigma_0 = I$, $\sigma_1 = X$, $\sigma_2 = Y$ and $\sigma_3 = Z$. We have seen $I, X, Z$ in class. The operator $Y$ is

   $$Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

   (Hint: start with $H = Z^{(1)} \otimes Z_{(2)} \otimes \cdots \otimes Z_{(n)}$.)

2. Suppose that a boolean function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ has $M$ solutions. That is, the number of $x$ such that $f(x) = 1$ is $M$. Prove that any quantum circuit which has the usual black-box access to $f$ requires $\Omega(\sqrt{N/M})$ queries to $f$ in the worst case to find a solution.