CS 264: Quantum Computation

Homework 3

Spring 2012

Due: May 21, 2012

1. Consider the quantum circuit that computes the Fourier transform mod 2^n. Now assume it is only necessary to compute the Fourier transform to within ϵ, where distance is measured in the operator norm. How much more efficient can you make the circuit? (Hint: consider omitting some of the phase shifts for small angles).

2. Let $a|q$ and $b|q$. What is the Fourier transform mod q of the uniform superposition of all $0 \leq x < q$ such that $a|x$ or $b|x$?

3. Give a circuit which will compute $(2|0\rangle\langle 0| - I)$ on n-qubits. You can use auxiliary qubits if you need to but these should be initialized to $|0\rangle$ and reset back to $|0\rangle$ at the output of the circuit.

4. Show how to implement the function $|x\rangle \rightarrow |x + 1 \pmod{2)^n}\rangle$ using the Fourier transform. What is the complexity (big-Oh of the number of gates) of your circuit?

5. Consider a function $f : \{0,1\}^n \rightarrow \{0,1\}^m$. Devise an quantum algorithm that will find $\min_y f(y)$. Your algorithm should compute the correct answer with probability at least 1/2. Express the complexity of your algorithm in terms of the number of gates in the circuit and the number of queries to f (although you need only express these in terms of the asymptotic dependency on m and n).

   You can assume the existence of a quantum algorithm for approximate counting. This algorithm will take a boolean function $g : \{0,1\}^n \rightarrow \{0,1\}$ given as a black box and will estimate the value of $M = |\{x \mid g(x) = 1\}|$. If $t = l + \lceil \log(2 + 1/(2\epsilon)) \rceil$, where l is a parameter denoting the accuracy in the estimate, the algorithm will use $O(t + n + 1)$ qubits. The query complexity is $\Theta(t)$ and the computational complexity is $\Theta(n + t)$. The algorithm will produce $\tilde{M}$ such that with probability at least $1 - \epsilon$,

$$|M - \tilde{M}| < \left(\sqrt{2MN} + \frac{N}{2^{l+1}}\right) 2^{-l}.$$