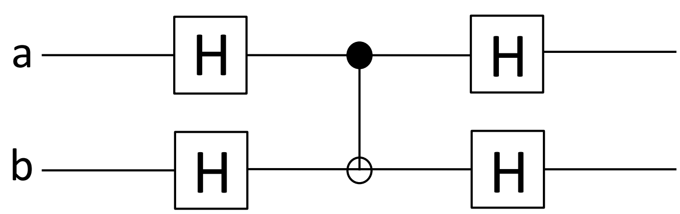
# CS 264: Quantum Computation
## Homework 2
## Spring 2012

Due: May 4, 2012

1. Prove that the Bell state $|\Psi^-\rangle = 1/\sqrt{2}(|01\rangle - |10\rangle)$ is rotationally invariant. (i.e. $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|vv^\perp\rangle - |v^\perp v\rangle)$ for any 1-qubit state $|v\rangle$).

2. Describe the action of a CNOT gate if the target bit is $|-\rangle$.

   Now show that the following circuit is effectively a CNOT gate with the control and target qubits swapped (i.e. $b$ is the control and $a$ is the target).




3. Prove that we can assume without loss of generality that all the amplitudes in a quantum computation are real. Prove that any quantum circuit with $m$ 2-qubit gates can be simulated by a quantum circtuit with $m$ 3-qubit gates in which the amplitdues of all the intermediate states computed by the circuit are real.

4. Prove the principle of deferred measurement: suppose a unitary operator $V \otimes I$ acts on a state $|\phi\rangle$ of $n+m$ qubits, where $V$ acts on the first $n$ qubits. Consider the distribution $\mathcal{D}$ that results from measuring the first $n$ qubits after $V \otimes I$ is applied. Prove that this is the same distribution that results from first measuring the last $m$ qubits, applying $V \otimes I$ and then measuring the first $n$ qubits.

5. Suppose that a 2-qubit state is shared by Alice and Bob. Suppose that Alice performs a unitary opertion $U$ on her qubit and then Bob measures his qubit in some basis $\{|\phi\rangle, |\phi^\perp\rangle\}$. Show that the results of Bob's measurement does not depend on the unitary operation chosen by Alice.