CS 264: Quantum Computation
Homework 1
Fall 2009

Due: April 20, 2012

1. Prove that the eigenvalues of a unitary operator can be written in the form $e^{i\theta}$ for some real θ.

2. Prove that two eigenvectors of a Hermitian matrix with different eigenvalues are orthogonal.

3. Prove that a normal matrix is Hermitian of and only if it has real eigenvalues.

4. The Hadamard operator on one qubit may be written as

$$H = \frac{1}{\sqrt{2}} \left[(|0\rangle + |1\rangle)\langle 0| + (|0\rangle - |1\rangle)\langle 1| \right].$$

   Give a closed form expression for $H^{\otimes n}$ using outer-bracket notation in the standard basis.

5. Consider the quantum state $|\psi\rangle = 1/\sqrt{2}(|0\rangle + e^{i\theta}|1\rangle)$. Describe a measurement that will yield some information about the phast θ so that if you are given many copies of $|\psi\rangle$ you can determine θ to arbitrary accuracy.

6. Suppose that A' and A'' are martix representations of linear operator A on a vector space V with respect to two different orthonormal bases $|v_i\rangle$ and $|w_i\rangle$. The elements of A' and A'' are $A'_{ij} = \langle v_i|A|v_j\rangle$ and $A''_{ij} = \langle w_i|A|w_j\rangle$. Characterize the relationship between A' and A''.