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10:01 AM

Unstructured Search

We have access to a function $f: \{0,1\}^n \to \{0,1\}$ through an oracle O_f:

$$O_f: |x\rangle |y\rangle \longrightarrow |x\rangle |y \oplus f(x)\rangle$$

($|x\rangle$: n qubits; $|y\rangle$: 1 qubit.)

We want to determine if $\exists\, a$ s.t. $f(a) = 1$.

Let $M = |\{x \mid f(x) = 1\}|$ $\quad M \geq 1$?

Last time we showed that if we are guaranteed that if $M = 0$ or $M = 1$ then there is a quantum circuit that can distinguish between those two cases using $O(\sqrt{N})$ queries to f. $(N = 2^n)$.

What if $|\{a \mid f(a) = 1\}| = M > 1$?

We will first show how to generalize Grover search to the situation where M is known.

Suppose you know $M \geq 1$ and want to find an input a such that $f(a) = 1$.

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$$|\phi_1\rangle = \frac{1}{\sqrt{M}} \sum_{\substack{x: \\ f(x)=1}} |x\rangle \qquad |\phi_0\rangle = \frac{1}{\sqrt{N-M}} \sum_{\substack{x: \\ f(x)=0}} |x\rangle$$

$U = e^{iH}$

$$|\psi\rangle = \sqrt{\frac{N-M}{N}}\,|\phi_0\rangle + \sqrt{\frac{M}{N}}\,|\phi_1\rangle$$

$$\sin\theta = \sqrt{\frac{M}{N}}$$

$$\sin\theta = |\langle\phi_1|\psi\rangle| = \sqrt{M/N}.$$

If we know M, can select # iterations c such that

$$(2c+1)\theta \approx \pi/2.$$

If $M \ll N$ $\quad \theta \approx \sqrt{M/N}$ $\quad$ # iterations is $O\left(\sqrt{\frac{N}{M}}\right)$

c = # iterations.

$$\frac{c}{\text{(const)}}\sqrt{\frac{M}{N}} \approx \text{const}$$

$$c \approx O\left(\sqrt{\frac{N}{M}}\right).$$

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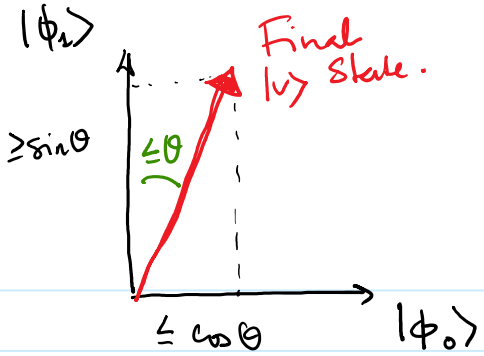

Error:

Since we advance by 2θ in each iteration, can select the # of iterations so that the final state is within θ of the target $|\phi_1\rangle$

The probability that we measure a such that $f(a)=1$ is:

$$|\langle v|\phi_1\rangle|^2 \geq \cos^2\theta$$

Probability of error is: $|\langle v|\phi_0\rangle|^2 \leq \sin^2\theta$

$$\sin^2\theta \approx \frac{M}{N}.$$

Suppose we want error $\leq 1/4$. $M < \frac{N}{4}$.

If $M > \frac{N}{4}$ Just randomly sample $f(x)$ on random x. Expected # trials needed to find a s.t. $f(a)=1$. is ≤ 4.

If $M \leq \frac{N}{4}$ $\sin^2\theta \leq \frac{1}{4}$.

* Still need to handle the case where M is unknown *