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Classical View of Nature:

- Object has inherent properties.
- measurement only reveals those properties.
- Properties exist independent of observation.

(A coin flip may appear random but the outcome is predictable given enough info about the pre-existing conditions.)

Quantum View:

- Some properties of an object are not inherent/fixed until measured.

Einstein, Podoolsky + Rosen (EPR) [1935] rejected this view of nature.
They conjecture that an outcome of a measurement would be predictable if only we had more information about the state.

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EPR suggested the thought experiment of measuring one qubit of an entangled pair:

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

After Alice measures her qubit, she can predict the outcome of a measurement of Bob' qubit w/ probability 1.

EPR assertion: If Alice can predict Bob's qubit, it should be included in reality (encoded somewhere in nature) before Alice's measurement.

This assertion was experimentally invalidated with a result known as Bell's inequality.

[John Bell 1964]

Perform a thought experiment + analyze the situation with the classical view and the quantum view.
Each approach will lead to a different outcome.
This can be validated experimentally.

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2 player game

Alice + Bob are separated so they can't communicate.

They each receive a random bit X_A for Alice, X_B for Bob. $X_A + X_B$ are independent.

Alice outputs bit a. Bob outputs bit b.

Goal: $X_A \wedge X_B = \underbrace{a \oplus b}_{a+b \bmod 2}$.

Classical Model:

Even if Alice and Bob meet before the start of the game and agree upon an arbitrarily long random string, (this would correspond to the "hidden information stored in the qubits).

it can be proven that the optimal strategy is for Bob + Alice to both output 0.

$a = b = 0$. Prob of success:

$\text{Prob}[X_A \wedge X_B = 0] = 3/4$.

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Quantum Mechanical Model:

Alice and Bob share an entangled pair of qubits:

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

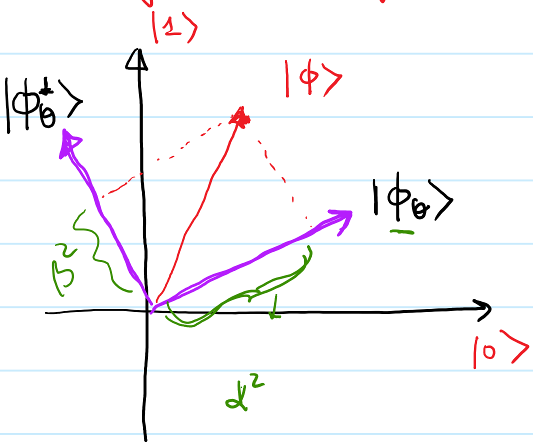
Alice has one qubit and Bob has the other qubit.

If the strange behavior of this entangled state can be explained by a hidden variable theory then every quantum protocol should have a classical counterpart.

Here is a quantum protocol that does strictly better than the classical protocol:

Define 1-qubit basis:

$$|\phi_\theta\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$
$$|\phi_\theta^\perp\rangle = \sin\theta\,|0\rangle - \cos\theta\,|1\rangle$$



"Measure in the θ basis" is well defined.

$|\phi_\theta\rangle \quad |\phi_\theta^\perp\rangle$

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$\theta = -\pi/16$

Alice + Bob share: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Each will measure their bit in some basis.

$X_A = 0$ Alice measures in $-\pi/16$ basis — θ — Output 0 if $|\phi_\theta\rangle$

$X_A = 1$ Alice measures in $3\pi/16$ basis.

$X_B = 0$ Bob measures in $\pi/16$ basis. — Output 1 if $|\phi_\theta^\perp\rangle$

$X_B = 1$ Bob measures in $-3\pi/16$ basis. — w.

$|\phi_\theta\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|\phi_\theta^\perp\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle$

$$\cos\theta|\phi_\theta\rangle + \sin\theta|\phi_\theta^\perp\rangle = \cos\theta(\cos\theta|0\rangle + \sin\theta|1\rangle) + \sin\theta(\sin\theta|0\rangle - \cos\theta|1\rangle)$$
$$= (\cos^2\theta + \sin^2)|0\rangle = |0\rangle$$

$$\sin\theta|\phi_\theta\rangle - \cos\theta|\phi_\theta^\perp\rangle = |1\rangle$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(\cos\theta|\phi_\theta\rangle + \sin\theta|\phi_\theta^\perp\rangle)|0\rangle + \frac{1}{\sqrt{2}}(\sin\theta|\phi_\theta\rangle - \cos\theta|\phi_\theta^\perp\rangle)|1\rangle$$

$$\frac{1}{\sqrt{2}}|\phi_\theta\rangle(\cos\theta|0\rangle + \sin\theta|1\rangle) + \frac{1}{\sqrt{2}}|\phi_\theta^\perp\rangle(\sin\theta|0\rangle - \cos\theta|1\rangle)$$
$$= \frac{1}{\sqrt{2}}(|\phi_\theta\rangle|\phi_\theta\rangle + |\phi_\theta^\perp\rangle|\phi_\theta^\perp\rangle)$$

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$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(\cos\theta|\phi_\theta\rangle + \sin\theta|\phi_\theta^\perp\rangle\right)|0\rangle + \frac{1}{\sqrt{2}}\left(\sin\theta|\phi_\theta\rangle - \cos\theta|\phi_\theta^\perp\rangle\right)|1\rangle$$

$$\frac{1}{\sqrt{2}}\left(\cos\theta|\phi_\theta\rangle|0\rangle + \sin\theta|\phi_\theta^\perp\rangle|0\rangle + \sin\theta|\phi_\theta\rangle|1\rangle - \cos\theta|\phi_\theta^\perp\rangle|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|\phi_\theta\rangle\left(\cos\theta|0\rangle + \sin\theta|1\rangle\right) + |\phi_\theta^\perp\rangle\left(\sin\theta|0\rangle - \cos\theta|1\rangle\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\phi_\theta\rangle|\phi_\theta\rangle + |\phi_\theta^\perp\rangle|\phi_\theta^\perp\rangle\right)$$

If we measure in $|\phi_\theta\rangle$ $|\phi_\theta^\perp\rangle$ basis then

Prob outcome $|\phi_\theta\rangle$: $1/2$

Prob outcome $|\phi_\theta^\perp\rangle$: $1/2$

$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

Alice measures in $|\phi_\theta\rangle$ $|\phi_\theta^\perp\rangle$ basis

$\xrightarrow{1/2}$ $|\phi_\theta\rangle|\phi_\theta\rangle$ $\qquad$ $\xrightarrow{1/2}$ $|\phi_\theta^\perp\rangle|\phi_\theta^\perp\rangle$

Bob measures in w basis

Prob?? $\qquad$ Prob??

$|\phi_w\rangle|\phi_w\rangle$ $\qquad$ $|\phi_w^\perp\rangle|\phi_w^\perp\rangle$ $\qquad$ $|\phi_w\rangle|\phi_w\rangle$ $\qquad$ $|\phi_w^\perp\rangle|\phi_w^\perp\rangle$

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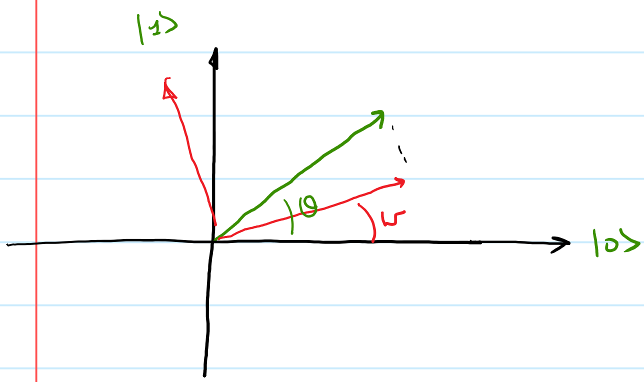
State $|\phi_\theta\rangle$ measure in $|\phi_w\rangle$ $|\phi_w^\perp\rangle$ basis:



Prob $|\phi_w\rangle = \cos^2(\theta - w)$.

$|\phi_w^\perp\rangle = \sin^2(\theta - w)$.

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Alice measures in $|\phi_\theta\rangle$ $|\phi_\theta^\perp\rangle$ basis

$a = 0$ (1/2): $|\phi_\theta\rangle|\phi_\theta\rangle$

$a = 1$ (1/2): $|\phi_\theta^\perp\rangle|\phi_\theta^\perp\rangle$

Bob measures in w basis

$b = 0$: $|\phi_w\rangle|\phi_w\rangle$ Prob $\cos^2(\theta - w)$ $a \oplus b = 0$

$b = 1$: $|\phi_w^\perp\rangle|\phi_w^\perp\rangle$ Prob $\sin^2(\theta - w)$ $a \oplus b = 1$

$b = 0$: $|\phi_w\rangle|\phi_w\rangle$ Prob $\sin^2(\theta - w)$ $a \oplus b = 1$

$b = 1$: $|\phi_w^\perp\rangle|\phi_w^\perp\rangle$ Prob $\cos^2(\theta - w)$ $a \oplus b = 0$

Prob 1/4

$\Rightarrow$ If $X_A = X_B = 0$ $X_A \wedge X_B = 0$ want $a \oplus b = 0$.

$\pi/16 - \pi/16$ $\theta = \pi/16$

Happens with prob $\cos^2(\theta - w)$ $w = -\pi/16$

$\cos^2(\pi/8) \sim .853$

If $X_A = X_B = 1$ $X_A \wedge X_B = 1$ want $a \oplus b = 1$. $\sin^2\left(\frac{3\pi}{8}\right)$

prob $\sin^2(\theta - w)$ $\Rightarrow \theta = 3\pi/16$ $\Rightarrow w = -3\pi/16$ $\sin^2\left(\frac{3\pi}{16} - -\frac{3\pi}{16}\right) \approx .85..$

Prob $\sin^2(\theta - \omega)$

$\Rightarrow \omega = -3\pi/16$

$\sin^2\left(\frac{3\pi}{16} - -\frac{3\pi}{16}\right) \approx .85$