Midterm Solutions

Monday, December 3, 2018 4:17 PM

Undergrad Version:

1). After CNOT : $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
After Z + X : $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.

2) Prob 0 : $\frac{1}{3}$ Resulting state: $|01\rangle$.

Prob 1 : $2/3$ Resulting state: $\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$.

3) $\alpha = \langle + | \phi \rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}} + i \frac{1}{\sqrt{2}}\sqrt{\frac{2}{3}}$

$= \frac{1}{\sqrt{6}} + i/\sqrt{3}$

$\beta = \langle - | \phi \rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}} - i\sqrt{\frac{2}{3}}\right) = \frac{1}{\sqrt{6}} - \frac{i}{\sqrt{3}}$

4) a) $\sum_{j=1}^{N} \lambda_j |v_j\rangle\langle v_j|$

b) $\sum_{j=1}^{N} (\lambda_j)^2 |v_j\rangle\langle v_j|$

Monday, December 3, 2018 4:21 PM

$x = 1111010001$

5 a) $\frac{1}{\sqrt{2^{10}}} \cdot (-1)^{x \cdot z} = \frac{1}{2^5}$ $z = 1110000001$

b) $\frac{1}{\sqrt{2^{10}}} (-1)^{x \cdot z} = \frac{-1}{2^5}$ $z = 0010000111$

6.
$$\begin{bmatrix} 0 & 1 & & & & & & \\ 1 & 0 & & & & 0 & & \\ & & 1 & 0 & & & & \\ & & 0 & 1 & & & & \\ & & & & 0 & 1 & & \\ & 0 & & & 1 & 0 & & \\ & & & & & & 0 & 1 \\ & & & & & & 1 & 0 \end{bmatrix}$$

7. $U|\phi_1\rangle$ and $U|\phi_2\rangle$ are orthogonal

because $\langle \phi_1 | \phi_2 \rangle = 0$ and U is unitary.

$U|\phi_1\rangle = |000 \cdots 0\rangle$

therefore if $U|\phi_2\rangle$ is expressed in standard basis, amplitude of $|0 \cdots 0\rangle$ is zero.

Measure. If ≥ 1 occurred, state was $|\phi_2\rangle$

If all 0's, state was $|\phi_1\rangle$.

Monday, December 3, 2018 4:26 PM

Grad Version

1. Same as #3 in ugrad test.

2. a & b Same as 5 a + b in ugrad test.

c) $\frac{(-1)^{x \cdot z}}{2^5}$ $\qquad x \cdot z = \left[\sum_{j=1}^{10} x_i \cdot z_i \right] \bmod 2$

3. Same as # 4 in Ugrad test.

4. $$\begin{bmatrix} 1 & 0 & & & & & & \\ 0 & 1 & & & & 0 & & \\ & & 0 & 1 & & & & \\ & & 1 & 0 & & & & \\ & & & & 1 & 0 & & \\ & & 0 & & 0 & 1 & & \\ & & & & & & 0 & 1 \\ & & & & & & 1 & 0 \end{bmatrix}$$

5 a) $\frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\beta}{\sqrt{2}} |100\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\beta}{\sqrt{2}} |111\rangle$

b) $\frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\beta}{\sqrt{2}} |110\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\beta}{\sqrt{2}} |101\rangle$

Monday, December 3, 2018 4:31 PM

$$\frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|101\rangle$$

c) $= \frac{\alpha}{2}\left(|000\rangle + |100\rangle + |011\rangle + |111\rangle\right)$

$$+ \frac{\beta}{2}\left(|010\rangle - |110\rangle + |001\rangle - |101\rangle\right)$$

d)

Outcome	Prob	Resulting State	Bob's Correction
00	$\frac{\lvert\alpha\rvert^2}{4} + \frac{\lvert\beta\rvert^2}{4} = \frac{1}{4}$	$\alpha\|000\rangle + \beta\|001\rangle$	Nothing
01	$1/4$	$\alpha\|011\rangle + \beta\|010\rangle$	X
10	$1/4$	$\alpha\|100\rangle - \beta\|101\rangle$	Z
11	$1/4$	$\alpha\|111\rangle - \beta\|110\rangle$	XZ

e)