1. *(10 points)* A directed graph G has five vertices numbered 1 through 5. The 5×5 matrix A is the matrix representation of the graph G. The matrices A^2 and A^3 are given below. Use this information about G to answer the following questions. For the yes/no questions, circle "Y" or "N".

$$A^2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(a) Write down Row 4 of A^5:

(b) List the vertices v such that there is a walk of length 3 from vertex v to vertex 2 in G.

(c) What is the out-degree of vertex 4 in the transitive closure of G (i.e., G^+)?

(d) Is there a walk of length 4 from vertex 4 to vertex 5 in G? (*Hint:* $A^2 \cdot A^2 = A^4$). Y N

(e) Is $(2, 2)$ in the transitive closure of G? Y N

(f) Is $(5, 3)$ an edge in G^3? Y N

(g) Is there a circuit of length 3 in G? Y N

2. *(3 points)* Define relations R_1, R_2, and R_3 as follows:

- $R_1 = \{(x, y) : x \leq y\}$
- $R_2 = \{(x, y) : x < y\}$
- $R_3 = \{(x, y) : x > y\}$

For the next three questions, your answer must be one of the following choices. Put the corresponding letter in the space provided.

A. R_1 B. R_2 C. R_3 D. $\mathbb{R} \times \mathbb{R}$

(a) $R_2 \circ R_1 =$ _____

(b) $R_3 \circ R_2 =$ _____

(c) $R_1 \circ R_1 =$ _____

8. *(10 points)* The players on a football team are all weighed on a scale. The scale rounds the weight of every player to the nearest pound. The number of pounds read off the scale for each player is called his *measured weight*. The domain for each of the following relations below is the set of players on the team. Select from the following options to describe each relation. Put the corresponding letter in the space provided.

A. Strict order but not a total order.
B. Strict order and a total order.
C. Partial order but not a total order.
D. Partial order and a total order.
E. Equivalence relation.
F. Neither a partial order, strict order nor an equivalence relation.

(a) Player x is related to player y if the measured weight of player x is at least the measured weight of player y. No two players have the same measured weight.

Answer_____

(b) Player x is related to player y if the measured weight of player x is at least the measured weight of player y. There are at least two players with the same measured weight and at least two players with different measured weights.

Answer_____

(c) Player x is related to player y if the measured weight of player x is more than the measured weight of player y. No two players have the same measured weight.

Answer_____

(d) Player x is related to player y if the measured weight of player x is more than the measured weight of player y. There are at least two players with the same measured weight and at least two players with different measured weights.

Answer_____

(e) Player x is related to player y if the measured weight of player x is equal to the measured weight of player y.

Answer_____

9. *(4 points)* Below is a database showing the daily train schedule for a train station.

Departure Time	Destination	Track	Local/Express
8:00AM	Brussels	1	Express
9:15AM	Helsinki	2	Local
9:22AM	Munich	1	Local
9:32AM	Brussels	2	Express
11:00AM	Amsterdam	1	Express
11:17AM	Brussels	2	Local
11:44AM	Amsterdam	2	Local

(a) What would be the result of the query SELECT[Destination="Brussels" and Track="2"]?

(b) What series of operations would you performed if you wanted to know the departure times of all the Express trains to Brussels?

10. *(8 points)* The relation R is a partial order on the set $\{A, B, C, D, E, F, G, H\}$.

$$R = \{(C,B),(D,C),(C,F),(G,H),(F,B),(A,B),(D,F),(D,B),(D,A),\\ (A,A),(B,B),(C,C),(D,D),(E,E),(F,F),(G,G),(H,H)\}$$

(a) Draw the Hasse diagram for this partial order.

(b) What are the minimal elements of the partial order?

(c) Circle the pairs of elements that are comparable.

- D and B
- G and B
- C and A
- C and D