| | | |
|---|---|---|
| Idempotent laws | $p \vee p \equiv p$ | $p \wedge p \equiv p$ |
| Associative laws | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ |
| Commutative laws | $p \vee q \equiv q \vee p$ | $p \wedge q \equiv q \wedge p$ |
| Distributive laws | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| Identity laws | $p \vee F \equiv p \qquad p \vee T \equiv T$ | $p \wedge F \equiv F \qquad p \wedge T \equiv p$ |
| Involution laws | $\neg\neg p \equiv p$ | |
| Complement laws | $\neg p \vee p \equiv T$ | $\neg p \wedge p \equiv F$ |
| DeMorgan's laws | $\neg(p \wedge q) \equiv \neg p \vee \neg q$ | $\neg(p \vee q) \equiv \neg p \wedge \neg q$ |
| Conditional identities | $p \rightarrow q \equiv \neg p \vee q$ | $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ |

1. *(4 points)* In each question below, two propositions are given which are logically equivalent. Give the name of the law that can be used to show that the two propositions are logically equivalent.

(a) $\neg((w \vee p) \wedge (\neg q \wedge \neg w))$
$\neg(w \vee p) \vee \neg(\neg q \wedge \neg w)$

(b) $(\neg p \wedge (r \vee \neg q)) \vee (\neg p \wedge w)$
$\neg p \wedge ((r \vee \neg q) \vee w)$

(c) $r \wedge (p \vee q)$
$r \wedge (q \vee p)$

(d) $(r \wedge p) \vee (\neg p \wedge q)$
$((r \wedge p) \vee \neg p) \wedge ((r \wedge p) \vee q)$

2. *(4 points)* Fill in the steps of the argument below to show that $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$. Each step is labeled with the name of a propositional law. Fill in each blank with a proposition so that each proposition can be obtained from the one before it by applying the stated law.

| | |
|---|---|
| $\neg(\neg p \wedge q) \wedge (p \vee q)$ | Start |
| | De Morgan's Law |
| | Double Negation Law |
| | Distributive Law |
| | Complement Law |
| $p$ | Identity Law |

3. *(6 points)* Suppose that the propostitional variables $p$, $q$, and $r$ have the following truth values:

$$p = \text{T} \qquad q = \text{T} \qquad r = \text{F}.$$

Circle the logical expressions that evaluate to true:

(a) $p \vee q$
(b) $q \wedge \neg r$
(c) $r \rightarrow q$
(d) $p \leftrightarrow (q \wedge r)$
(e) $(q \wedge \neg r) \rightarrow \neg p$
(f) $\neg(p \vee q \vee r)$

4. *(3 points)* Show that $(p \leftrightarrow \neg q) \rightarrow \neg p$ is not a tautology. If you use a truth table, you need to be specific about which row of the truth table justifies your answer.

5. *(8 points)* Define the propositional variable:

   - $b$: Joe has maintained a B average.
   - $c$: Joe has received below a C in a class.
   - $h$: Joe is eligible for the honors program.

   Translate the following sentences into logical propositions. For example the statement: ”If Joe has maintained an B average, then he is eligible for the honors program” would be translated as $b \rightarrow h$.

   (a) Maintaining a B average is sufficient for Joe to be eligible for the honors program.

   (b) Joe is eligible for the honors program, only if he has maintained a B average.

   (c) Joe has maintained a B average even though he did receive a grade below a C in a class.

   (d) Joe is eligible for the honors program if and only if he has maintained a B average and has not received below a C in a class.

6. *(3 points)* Consider the conditional statement:

   ”If Joe maintained a B average, then he is eligible for the honors program.”

   Label each statement below according to whether it is the contrapositive, converse or inverse of the statement above.

   (a) If Joe is not eligible for the honors program then he did not maintain a B average.

   (b) If Joe is eligible for the honors program, then he maintained a B average.

   (c) If Joe did not maintain a B average, then he is not eligible for the honors program.

7. *(12 points)* A student club holds a meeting. The predicate $M(x)$ denotes whether person $x$ came to the meeting on time. The predicate $O(x)$ refers to whether person $x$ is an officer of the club. The predicate $D(x)$ indicates whether person $x$ has paid his or her club dues. The domain is the set of all members of the club. Give a logical expression that is equivalent to each English statement below.

   (a) Someone is not an officer.

   (b) All the officers came on time to the meeting.

   (c) Everyone was on time for the meeting.

   (d) Everyone paid their dues or came on time to the meeting.

   (e) At least one person paid their dues and came on time to the meeting.

   (f) There is an officer who did not come on time for the meeting.

8. *(10 points)* This question refers to the same club and predicates in the previous question. Suppose that the club has five members. The names of the members and their truth values for each of the predicates is given in the table below. Circle the expressions that are true for this particular club. If a universal statement is not true, give a counter-example. If an existential statement is true, give an example.

| Name | $M(x)$ | $O(x)$ | $D(x)$ |
|---|---|---|---|
| Hillary | T | F | T |
| Bernie | F | T | F |
| Donald | F | T | F |
| Jeb | F | T | T |
| Carly | F | T | F |

   (a) $\forall x\ \neg(O(x) \leftrightarrow D(x))$

   (b) $\forall x((x \neq \text{Jeb}) \rightarrow \neg(O(x) \leftrightarrow D(x))$

   (c) $\forall x(\neg O(x) \rightarrow D(x))$

   (d) $\exists x(M(x) \wedge D(x))$

   (e) $\forall x(M(x) \vee O(x) \vee D(x))$

   (f) $\forall x\ \neg D(x)$

   (g) $M(\text{Jeb}) \wedge D(\text{Hillary})$

   (h) $D(\text{Bernie}) \rightarrow O(\text{Bernie})$

   (i) $\exists x(O(x) \rightarrow M(x))$

   (j) $\exists x(M(x) \wedge O(x) \wedge D(x))$