ICS 6B **Due: Wednesday, November 4, 2015, 5:30PM**

Homework 5

Instructor: Sandy Irani

Covers Sections 4.4-4.5, 6.1-6.2

Please make sure to staple together all the pages of your written homework before putting it in the ICS 6B slot. Also write your student ID number and your name very clearly in the upper right corner of every page. The written portion should be turned into the dropbox labeled "ICS 6B" on the first floor of Bren Hall.

1. For each function defined in problems 10, 11 and 12 from Homework 4, of the function has an inverse, give its inverse.

2. Define the following functions:

   - $f : \{0,1\}^3 \rightarrow \{0,1\}^3$. The output of f is obtained by swapping the first and the last bits. For example $f(110) = 011$.
   - $g : \{0,1\}^3 \rightarrow \{0,1\}^4$. The output of g is obtained by adding an extra copy of the first bit to the end of the string. For example $g(011) = 0110$.
   - $h : \{0,1\}^3 \rightarrow \{0,1\}^3$. The output of h is obtained by replacing the last bit with a copy of the first bit. For example $h(011) = 010$.

   (a) What is $g \circ f(110)$?

   (b) What is $h \circ f(110)$?

   (c) What is $g \circ h(110)$?

   (d) What is the target of $g \circ f$?

   (e) What is the range of $g \circ h$?

3. Consider functions f, g and h whose domain and target are $\mathbb{Z}^+$.

   - $f(x) = x^2$
   - $g(x) = \lfloor \log_2 x \rfloor$
   - $h(x) = \lceil x/5 \rceil$

   (a) Evaluate $f \circ h(7)$.

   (b) Evaluate $g \circ h(114)$.

   (c) Evaluate $f \circ g \circ h(17)$.

   (d) What is the range of $g \circ h$?

   (e) What is the range of $f \circ g$?

4. Define the set $A = \{\text{r}, \text{o}, \text{t}, \text{p}, \text{c}\}$ and $B = \{\text{discrete}, \text{math}, \text{proof}, \text{proposition}\}$. Define the relation $R \subseteq A \times B$ such that $(letter, word)$ is in the relation if that letter occurs somewhere in the word.

   (a) Give the arrow diagran for the relation.

   (b) Give the matrix representation for the relation.

(c) Express the relation as a set of ordered pairs using roster notation.

5. Consider a binary relation whose domain is $\{0,1\}^3$. For $x, y \in \{0,1\}^3$, (x, y) is in the relation if the following condition is true:

   - For every $i = 1, 2, 3$, if the i^{th} bit of x is 1, then the i^{th} bit of y is also 1.

   (a) Is $(010, 100)$ in the relation? Why or why not?

   (b) Draw the arrow diagram for this relation.

   (c) Which of the following properties does the relation have?

   - Reflexive, anti-reflexive, symmetric, anti-symmetric, transitive.

6. For each of the relations below, answer the following three questions:

   - Is the relation reflexive, anti-reflexive or neither? If the answer is neither, give examples to justify your answer.
   - Is the relation symmetric, anti-symmetric or neither? If the answer is neither, give examples to justify your answer.
   - Is the relation transitive? If the answer is no, give an example to justify your answer.

   (a) The domain is a set of people. (x, y) is in the relation if person x is taller than person y. You can assume that the group contains at least two people who do not have the same height.

   (b) The domain is a set of people. (x, y) is in the relation if person x is a first cousin of person y (i.e., a parent of person x is a sibling of a parent of person y. You can assume that siblings do not have any children together. You can also assume that the relation is not empty, so there is at least one pair (x, y) suc that x is the first cousin of y.

   (c) The domain is a set of students at a university. (x, y) is in the relation if person x knows the student ID number for person y. You can assume that every student knows his or her own student ID number.

   (d) The domain is the set of real numbers. (x, y) is in the relation if $x + y = 0$.

   (e) The domain is the set of real numbers. (x, y) is in the relation if $x = 2y$.

   (f) The domain is the set of real numbers. (x, y) is in the relation if $x - y$ is a rational number.

   (g) The domain is the set of real numbers. (x, y) is in the relation if $x \neq y$.

   (h) The domain is $A = \{a, b, c, d\}$. The relation is $\{(a, b), (a, a), (b, b), (b, a), (c, d), (d, c)\}$.

   (i) The domain is the set $\{0,1\}^4$. For $x, y \in \{0,1\}^4$, (x, y) is in the relation if y can be obtained from x by flipping exactly one bit. Flipping a bit means changing to from 0 to 1 or from 1 to 0. For example $(0110, 1110)$ is in the relation because flipping the first bit of 0110 results in string 1110.

   (j) The domain is the set $\{0,1\}^4$. For $x, y \in \{0,1\}^4$, (x, y) is in the relation if y is obtained by reversing x. For example $(1110, 0111)$ is in the relation.

7. Give an example of a relation on the set $\{1, 2, 3\}$ that is neither reflexive nor anti-reflexive.

8. Is it possible to have a relation on a set that is symmetric and anti-symmetric? If so, give an example.