ICS H22 Homework 3

Fall 2004

Due: 3:00PM October 20, 2004

1. Bovine Businesses, Moo Marketing, Cow Company, and Udder United, hired the services of Big-Oh Analysis Inc. Big-Oh's analysts discovered that the cumulative earnings of each company could be described as a function, $\mathsf{e(t)}$ in terms of t, the number of quarters.
   Reminder: The log function is assumed to be of base 2 unless otherwise noted.

   These functions are:
   - Bovine Businesses: $\mathsf{e(t)} = 103t + 321t^{-2} + 750$
   - Moo Marketing: $\mathsf{e(t)} = 23t^2 + 18t$
   - Cow Company: $\mathsf{e(t)} = 630 \log t + 27$
   - Udder United: $\mathsf{e(t)} = 1.3^t - .3t^4$

   a) After 3 quarters, which company will have the highest earnings?
   b) After 6 quarters, which company will have the highest earnings?
   c) After 9 quarters, which company will have the highest earnings?
   d) After 50 quarters, which company will have the highest earnings?
   e) What is the earnings of each of the companies in Big-Oh notation?
   f) Given the ONLY Big-Oh for the earnings of each company, which company would you guess would earn the most after 3 quarters? What does this tell you about the shortcomings of Big-Oh analysis?

2. In this problem, n is a positive integer.

   Give a big-Oh characterization, in terms of n, of the running time of the algorithms Loop1(), Loop2(), Loop3(), Loop4(), and Loop5(), shown below.

   a)
   ```
   Algorithm Loop1(n):
     s ← 0
     for i ← 1 to n do
       s ← s + 1
   ```

   b)
   ```
   Algorithm Loop2(n):
     p ← 1
     for i ← 1 to n^2 do
       p ← p · i
   ```

   c)
   ```
   Algorithm Loop3(n):
     s ← 0
     for i ← 1 to 2n do
       for j ← 1 to i do
         s ← s + i
   ```

d) **Algorithm** Loop4(n):
$r \leftarrow 0$
while $r^2 < n$ **do**
$r \leftarrow r + 1$

e) **Algorithm** Loop5(X):
$r \leftarrow 1$
while $r < n$ **do**
$r \leftarrow r * 2$

3. For the typical algorithm you use to perform calculations by hand, determine the running time to:
   (a) Add two n-digit integers.
   (b) Multiply two n-digit integers.

4. An algorithm takes 0.5 ms for input size 100. How long will it take for an input size of 500 (assuming low-order terms are negligible) if the running time is
   (a) linear
   (b) logarithmic
   (c) quadratic
   (d) cubic

5. Is 2^{n+1} $O(2^n)$? Is 2^{2n} $O(2^n)$? In either case, if the answer is yes, give a proof.
6. Prove that $100n^2+203n+54$ is $O(n^2)$ by showing constants c and n_0 from the definition of 'big-Oh'.
7. Order the following functions by their asymptotic complexity. Indicate if there are functions which have the same growth rate.

$$\begin{aligned} f_1(n) &= 26n^3 \\ f_2(n) &= (.001)n \log n \\ f_3(n) &= 3^{n+9} \\ f_4(n) &= n/\log n \\ f_5(n) &= 17 \\ f_6(n) &= 57n \log(n^3) \\ f_7(n) &= 33n^2 + 301n \log^2 n \\ f_8(n) &= 3^{3n} \\ f_9(n) &= 16n \end{aligned}$$

Extra Credit:

- Find a linear time algorithm for the maxiumum subsequence sum problem discussed in class.
- An evil king has a cellar containing n bottles of expensive wine, and his guards have just caught a spy trying to poison the king's wine. Fortunately, the guards caught the

spy after he succeeded in poisoning only one bottle. Unfortunately, they don't know which one. To make matters worse, the poison the spy used was very deadly; just one drop diluted even a billion to one will kill someone. Even so, the poison works slowly; it takes a full month for the person to die. The king, of course, has taste testers, but he has only $\log n$ of them. Design an algorithm that allows the evil king to determine exactly which one of his wine bottles was poisoned in just one month's time. To simplify, assume that n is a power of 2, i.e., $n = 2^k$ for some integer k.