

Quiz 5

November 19

Instructor: Sandy Irani

1. (4 points) Is the following proof a constructive or non-constructive existence proof? Please explain why.

Theorem 1 *Given that $\sqrt{2}$ is an irrational number, show that there exists two irrational numbers x and y such that x^y is rational.*

Proof. Consider the number $z = \sqrt{2}^{\sqrt{2}}$. If z is rational, then we can choose $x = y = \sqrt{2}$ and the theorem holds. If z is irrational, consider $z^{\sqrt{2}}$. We know that

$$z^{\sqrt{2}} = \left(\sqrt{2}^{\sqrt{2}} \right)^{\sqrt{2}} = \sqrt{2}^{(\sqrt{2} * \sqrt{2})} = \sqrt{2}^2 = 2$$

Since 2 is rational, we can set $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ and the theorem holds.

2. (6 points) Prove that if x is rational and y is irrational, then $x + y$ is irrational.