

ICS 6A
Midterm Exam
Fall 2003

Instructor: Sandy Irani
Wednesday, November 5, 9:00AM

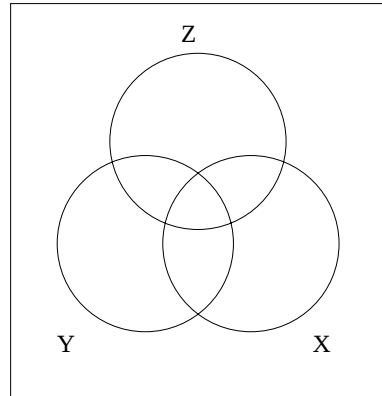
No Calculators are allowed in this exam. Complete all of the following questions. There are a total of 100 points. You can leave your answers in the form of an arithmetic expression unless the question specifies otherwise.

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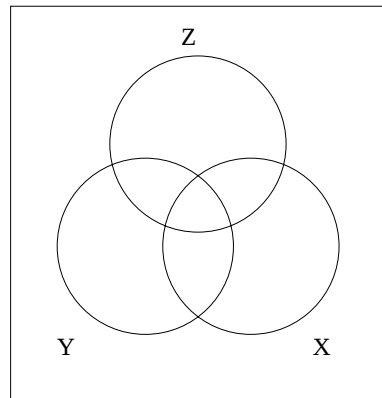
1. A college professor would like to select some of her graduate students to assist her in a research project. She will select a subset of her eight graduate students for the task.
 - (a) (*5 points*). How many ways are there for her to select the set of graduate students if she would like at least one student helping her with the project?
 - (b) (*5 points*). How many ways are there for her to select the set of students if there is one particular graduate student, say George, who she would like to have assisting her with the project (along with possibly others)?
 - (c) (*5 points*). How many ways are there for her to select the set of students if she would like exactly three students helping her with the project?
2. (*15 points*). Fourteen men and fifteen women are auditioning for a play. There are three different female roles in the play and four different male roles in the play. (Only women can play the female roles and only men can play the male roles). Each actor can play at most one part in the play. Note that actors and roles are distinct, so which actor gets which role makes a difference. How many ways are there to select the cast?

3. Fill in the Venn Diagram for the following sets. Make sure to make it clear what portions are included in the set.

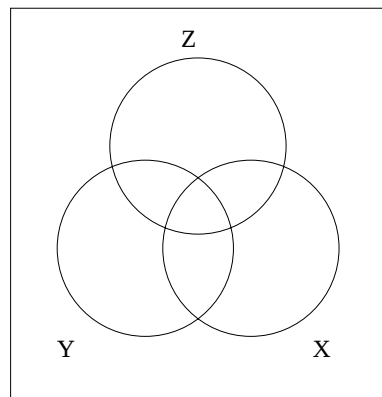
a) (5 points). $(Y - Z) \cup X$.



b) (5 points). $(Y \cup Z) - (Y \cup X)$.



c) (5 points). $(Y - Z) \cup (Y - X)$.



4. The local Chinese restaurant receives an order for fifteen lunch specials. There are seven different varieties of lunch specials on the menu.

(a) (*10 points*). How many distinct possibilities are there for the order? For example, one possibility is: 5 Mu-Shu Pork, 3 Kung Pao Chicken, and 7 Mongolian Beef.

(b) (*10 points*). How many distinct possibilities are there if at least one of each variety is ordered?

5. Suppose that Joe is dealt a hand of five cards from a perfectly shuffled deck.

(a) (*5 points*). What is the probability that he has at least one ace in his hand?

(b) (*10 points*). Suppose that Sally informs us that there are no hearts in the hand. What is the probability that Joe has a full house?

6. Let $t(x)$, $u(x)$ and $s(x)$ denote the following open statements:

$$t(x) : (x - 3)(x - 5) = 0$$

$$u(x) : x \text{ is odd}$$

$$s(x) : x > 0$$

For the universe of all integers, determine the truth or falsity of each of the following statements. Briefly justify your answer.

(a) (5 points). $\forall x[\neg t(x) \rightarrow \neg u(x)]$.

(b) (5 points). $\exists x[s(x) \wedge t(x)]$.

(c) (5 points). $\forall x[(t(x) \vee u(x)) \rightarrow s(x)]$.

(d) (5 points). $\forall x[(t(x) \wedge u(x)) \rightarrow s(x)]$.

7. (Extra credit: 3 points). How many positive integers are there less than one million whose digits add up to 20?