ICS 6 A
Final Exam
Fall 2003

Instructor: Sandy Irani

Wednesday, December 10, 8:00AM

No Calculators are allowed in this exam. Complete all of the following questions. There are a total of 100 points. You can leave your answers in the form of an arithmetic expression unless the question specifies otherwise.

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Total	

1. *(6 points)* Let the universe set be $U = \{1, 2, 3, 4, 5, 6, 7\}$. Let $A = \{1, 2, 3\}$, $B = \{3, 1, 5\}$, $C = \{4, 2, 5\}$ and $D = \emptyset$. Express each of the following sets given below:

   (a) $(A \cup B) - C$.

   (b) $\overline{B \cup C}$.

   (c) $(A \cup B \cup C) \cap D$.

2. *(4 points)* Consider the procedure given below. You can assume that n is a power of 2. What is the value of s at the end as a function of n?

$s \leftarrow 0$
while $(n > 1)$
    $n \leftarrow n/2$
    $s \leftarrow s + 1$

3. A convenience store offers seven different flavors of slurpies.

   (a) *(10 points)* Sam, Larry and Joe decide to each order a slurpy.

      i. How many different ways are there for them to place their order. Note that it matters who gets which flavor, so the choice where Sam gets cherry and Larry and Joe get grape is different from the choice where Larry gets cherry and Sam and Joe get grape.

      ii. What is the answer to the previous question if Sam, Larry and Joe each order different flavors?

   (b) *(10 points)* Sally walks in to the convenience store and places an order for fifteen slurpies.

      i. How many distinct possibilities are there for the order? For example, one possibility is: 5 cherry, 3 grape, 4 orange and 3 lime.

      ii. Sally decides she would like at least 5 cherry slurpies. Now how many distinct possibilities are there for the order?

4. *(10 points)* Let $t(x)$, $u(x)$ and $s(x)$ denote the following open statements:

$$\begin{aligned} t(x):&\quad (x-3)(x+3)=9 \\ u(x):&\quad x \text{ is odd} \\ s(x):&\quad x>0 \end{aligned}$$

For the universe of all integers, determine the truth or falsity of each of the following statements. Briefly justify your answer.

(a) $\forall x[\neg t(x) \rightarrow \neg u(x)]$.

(b) $\exists x[s(x) \wedge t(x)]$.

(c) $\forall x[(t(x) \vee s(x)) \rightarrow u(x)]$.

(d) $\forall x[(t(x) \wedge s(x)) \rightarrow u(x)]$.

5. *(10 points)* Consider the linear nonhomogeneous recurrence relation

$$a_n = -2a_{n-1} + 4a_{n-2} + 8a_{n-3} + F(n)$$

For each possibility of $F(n)$ below, give the general form of the particular solution to this recurrence relation. (Hint: $r^3 + 2r^2 - 4r - 8 = (r-2)(r+2)^2$.)

(a) $F(n) = 2$.

(b) $F(n) = 3n^2$.

(c) $F(n) = 4n3^n$.

(d) $F(n) = 7 \cdot 2^n$.

(e) $F(n) = 3n^3(-2)^n$.

6. Cheryl is dealt a five card hand from a standard deck of cards. You can assume the deck is perfectly shuffled so that each five-card hand is equally likely.

   (a) *(5 points)* What is the probability that she has exactly three hearts in her hand?

   (b) *(5 points)* Let X be a random variable denoting the number of hearts she has in her hand. What is the expected value of X?

   (c) *(5 points)* What if George peeks at her hand and tells us that she has no spades in her hand. Now what is the probability that she has exactly three hearts?

7. *(10 points)* Solve the following recurrence relation:

$$
\begin{aligned}
a_0 &= 1 \\
a_1 &= 9 \\
a_n &= 6 \cdot a_{n-1} - 9 \cdot a_{n-2} \quad \text{for } n > 1
\end{aligned}
$$

Express your answer as a closed form solution which is a function of n.

8. *(10 points)* Prove that if x is rational and y is irrational, then $x + y$ is irrational.

9. *(5 points)* Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0's.

10. *(10 points)* Use mathematical induction to show that 3 divides $n^3 + 2n$ whenever n is a nonnegative integer.