Due date: March 8 at 7:30 AM. You will need to submit this via GradeScope.

Instructions specific to this problem set: each of these questions requires that you build a graph that will solve this problem. A complete answer to these, or to questions like these, is to describe, in words, the graph you would build *for any instance of this problem* – not just a sample input. Once the graph is built, you will do something with the graph, such as calling an algorithm from lecture on the graph and interpreting its output. See your lecture notes from weeks 7 and 8 for examples. Then, describe the running time of your approach – how long would it take, given an input instance of the problem, to build the graph? How long does the algorithm you call take to run on a graph of the size you created?

1. Suppose we again have an $n \times n$ grid of squares, with each square being labeled with a positive integer. We start with two tokens, each on a *distinct* square. The goal is going to be to swap the positions of the two tokens. In any given turn, you may select any given token and move it up, down, left, or right by *a number of squares equal to the value of the square the other token is currently on.* For example, if the gold token is on a square labeled 3, then you may move the blue token up, down, left, or right by three squares.

   Under no conditions are you ever allowed to move a token off the grid, nor are you allowed to have the two tokens sitting on the same square.

   Your goal is to find the minimum number of moves required to swap the tokens for a given such puzzle, or to correctly report that the puzzle has no solution.

   For example, the following puzzle can be solved with five moves if gold is in the top-left and blue is in the bottom right. Take a moment to think about it before reading the solution that applies to this puzzle, listed below.

1	2	4	3
3	4	1	2
3	1	2	3
2	3	1	2

   First, move the gold token down. Because blue is on a two, gold moves down two. Then, move blue up by three (as gold is currently on a three). Gold right, blue left, gold down finishes the five move swap sequence.

2. Consider the problem A.J. Brown faces when driving to work each morning. This problem can be represented by a directed graph $G = (V, E)$ with a distinguished vertex s to represent A.J. Brown's house and another t to represent his destination. A subset of the vertices $C \subset V$ are marked as "Chipotle." A.J. wants to find the shortest path from his house to his destination such that he crosses *at most* one Chipotle. That is, we want the shortest path from s to t that uses at most one vertex from set C. You may assume that at least one such path exists. For full credit, your approach should have running time linear in the size of the original input graph.

   Hint: you are allowed to create a second graph, with approximately twice as many vertices as the input graph. What are the vertices in the new graph? What does being at any particular vertex during a path from s to t mean? How can you create the graph? What can you do once you create the graph?