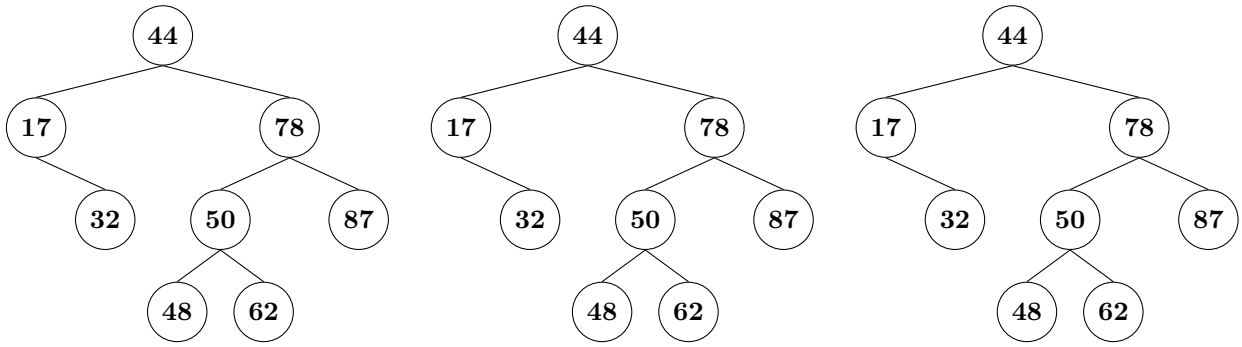
A **rooted tree** T is a set of **node**s storing elements in a **parent-child** relationship with the following properties:

- If T is nonempty, it has a designated root node, which has no parent.
- Every node v other than the root has a unique **parent**
- Every node with parent w is a **child** of w.

A binary search tree is a type of rooted tree. If the tree is non-empty, then the first node is a **root** of the tree (drawn at the top for some reason). You might think of a binary search tree like a linked list with two "next" pointers: one for the list that has only smaller values and one for only larger values. We call those "left" (smaller) and "right" (larger) pointers in this context.

Question 1. Starting with an initially empty binary search tree, insert the keys 50, 25, 75, 60, 55, 90, 65, 37

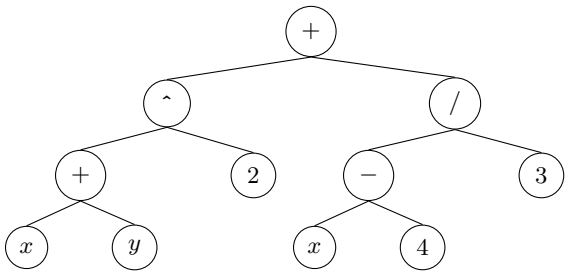
Question 2. From the following tree, what is the result if we delete node 62? What if we instead deleted 17? 78?



Question 3. Write a function that will output each key of a binary search tree from smallest to largest. You may assume you start with a pointer to the root.

Question 4. Write a function that will output each key of a binary search tree. Do so in such an order that if I insert each element of your output into an initially empty binary search tree, the result will be a tree that matches the tree we started with.

Question 5. Write code to evaluate a mathematical expression, represented as a *syntax tree*, such as the one below. A syntax tree will be a binary tree, but not a binary *search* tree. You may assume a function `eval(operator, left, right)` that evaluates any mathematical operator and `lookup(variable)` that looks up the value of a variable. You may also assume that if you call `lookup` with a constant as a parameter, it returns that constant.



Question 6. I have a binary (**not** a binary search tree) with the following properties:

- Each non-null[1] node of T contains a single character
- An in-order traversal of the tree reads "H, X, B, W, M, J, G, S, R, Y, P, T"
- A post-order traversal of the tree reads "H, X, W, M, J, B, S, R, T, P, Y, G"
  This is a reminder that this is not a binary search tree.

Draw the tree. Briefly explain (1-2 sentences) how you decided that is the tree that meets this criteria.

Reinforcement

After the first lecture, you should be able to define and use the following terms with respect to a binary search tree: parent, child, sibling nodes, leaf node, internal node, in-order traversal, pre-order traversal, post-order traversal. You should be able to describe the procedure to insert a new key into a binary search tree, how to delete from a binary search tree, and how to search a binary search tree for a particular key. Given a list of keys, you should be able to insert them into an initially empty binary search tree.

[1] Depending which textbook you are reading, this either means "all the internal nodes" or "all the internal nodes AND all the leaf nodes," so I am trying to be less ambiguous by saying every non-null node.