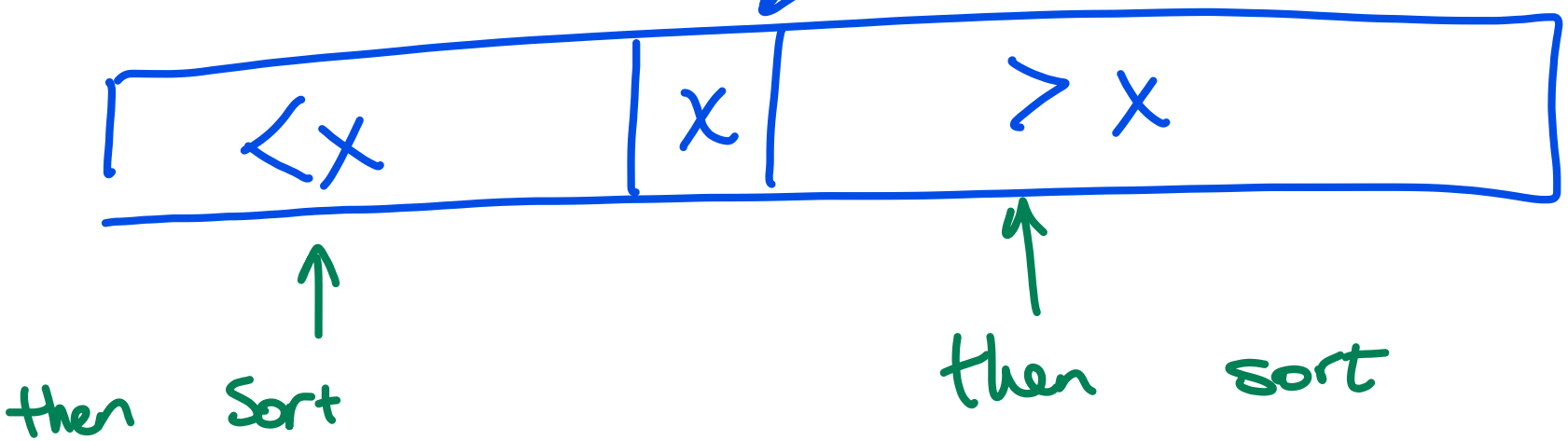
I & C SCI 46 Winter 2023
Lecture 23: QuickSort

QuickSort Step 1: Partition

48	41	51	66	84	89	87	68	37	23	96	98	52	14	62	43

1. Choose a pivot. x
2. Place that pivot in the right spot.
3. Pivot the rest of the array.



QuickSort

48	41	51	66	84	89	87	68	37	23	96	98	52	14	62	43
43 ~~89~~		14	23	37	48			84	66				51		89

Running time of QuickSort

- What is the worst-case running time?

  $\Theta(n^2)$

- Can you produce the worst-case running time?

  pivot min/max

- So why is it called "QuickSort" then?

Selecting a pivot

Variations of QuickSort change based on pivot selection:

- Deterministic, single-spot.

  good: easy to compute

  bad: easy to have worst case

- Uniformly at random

- Median-of-3

Average Case Analysis of QuickSort

Suppose

- All permutations equally likely
- All n values are distinct (for simplicity)
- Define $S_1, S_2, \ldots S_n$ as sorted order.

Let $P_{i,j}$ be probability we compare S_i and S_j.

$$P_{ij} = \frac{\#\text{yes}}{\#\text{total}} = \frac{2}{|j-i|+1}$$

Expected number of comparisons

$$E\Big(\sum_{i=1}^{n}\sum_{j=i+1}^{n} X_{i,j}\Big) = \sum_{i=1}^{n}\sum_{j=i+1}^{n} E(X_{i,j})$$

$$= \sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{2}{j-i+1}$$

$$= \sum_{i=1}^{n}\sum_{k=2}^{n-i+1} \frac{2}{k}$$

$$< \sum_{i=1}^{n}\sum_{k=1}^{n} \frac{2}{k}$$

i.r.v.: are i,j compared?

re-index

sum (inner) is $\Theta(\log n)$

is $\Theta(n \log n)$

Hybrid: Tim Sort

- The fastest sort algorithms are recursive
- Problem: recursion has an overhead

- Solution: