Due date: October 10 at 7:30 AM. You will need to submit this via GradeScope. Please refer to Problem Set 1 for the requirements of submitting a problem set.

1. An undirected graph is called d-regular if it is a simple graph and, for every vertex v, $\delta(v) = d$. For example, in a 3-regular graph, every vertex has three neighbors.
   (a) Either draw a 3-regular graph that has exactly nine (9) vertices, or explain why it cannot be done.
   (b) Either draw a 3-regular graph that has exactly twelve (12) vertices, or explain why it cannot be done.
   (c) Suppose I have a d-regular graph and I want to find a path that contains $d+1$ (or more) vertices. It turns out this is easy to do: pick an arbitrary start vertex. Until I have a path with $d+1$ vertices, pick an adjacent vertex to my current one that I have not yet visited. Add that edge to my growing path and set my current vertex to that one.

       Explain in 1-2 sentences why this will always produce a path with at least $d+1$ vertices. How do I know I won't "get stuck" at a vertex until it is at least the $d+1$th vertex I visit?

2. In lecture, we saw the number maze problem. I suggest you review your notes about that lecture before continuing with this homework. Suppose we again have an $n \times n$ grid of squares, with each square being labeled with a positive integer. We start with two tokens, each on a *distinct* square. The goal is going to be to swap the positions of the two tokens. In any given turn, you may select any given token and move it up, down, left, or right by *a number of squares equal to the value of the square the other token is currently on.* For example, if the gold token is on a square labeled 3, then you may move the blue token up, down, left, or right by three squares.

   Under no conditions are you ever allowed to move a token off the grid, nor are you allowed to have the two tokens sitting on the same square.

   Your goal is to find the minimum number of moves required to swap the tokens for a given such puzzle, or to correctly report that the puzzle has no solution. Describe how to use a graph to solve this problem. A complete answer includes a description of how to form a graph from an arbitrary puzzle as well as how to find a solution to the puzzle using the graph, or to report that none is possible.

   For example, the following puzzle can be solved with five moves if gold is in the top-left and blue is in the bottom right. Take a moment to think about it before reading the solution that applies to this puzzle, listed below.

1	2	4	3
3	4	1	2
3	1	2	3
2	3	1	2

   First, move the gold token down. Because blue is on a two, gold moves down two. Then, move blue up by three (as gold is currently on a three). Gold right, blue left, gold down finishes the five move swap sequence.

3. *You will probably want to wait until after lecture on October 7 before attempting this problem.* Suppose we have a counter that stores an arbitrary number of bits and counts in binary. It always begin at 0. The only mutator operation it can perform is to increment, adding one to the current count. This changes one or more bits. **Show** that if we start at 0 and perform k increment operations, a total of $\mathcal{O}(k)$ bits will change. Correct answers will reason about the **asymptotic** behavior of the total number of bit changes.

   Hint: There are multiple ways to approach this problem. One approach could use a credit argument, and you should refer to the array expansion of array-based Stacks and Queues for inspiration here. Another approach is numerical analysis. Try to find patterns in the total number of bit changes as k increases and inductively reason about the overall asymptotic behavior.

Additionally, our textbook of Goodrich and Tamassia has excellent practice problems available. Problem sets 2 and 3, and their associated lectures, collectively covered approximately chapters 4, 9.1, 9.2, and chapter 13. Note that some of the following problems relate to topics we will not have covered in lecture when this is due.

If you want additional practice problems, consider the following ones:R-9.1, R-9.7, R-9.8, R-9.9, R-9.15, C-9.6, C-9.7, C-9.11, C-9.14, R-13.1, R-13.2, R-13.3, R-13.4, R-13.5, R-13.7, R-13.8, R-13.9, R-13.11, R-13.12, R-13.13, R-13.14, R-13.15, R-13.16, R-13.31, R-13.32, C-13.2, C-13.7, C-13.8, C-13.10, C-13.23

After we discuss Minimum Spanning Trees in lecture, R-13.17, R-13.19, R-13.33, C-13.17 and C-13.27 are worth doing.

If you want practice problems related to $\mathcal{O}$ notation, consider R-4.6, R-4.7, R-4.8, R-4.10, R-4.11, R-4.13, R-4.16 through R-4.20, C-4.1, C-4.4, C-4.7, C-4.12 (this is more challenging than it looks), C-4.23, C-4.24