The first problem has appeared on exams in the past. The second was one of my favorite homework problems in the past (both as a student and as a professor), but I now think it works better as an in-class exercise.

1. Suppose we have a connected graph $G = (V, E)$, with n vertices and m edges, and distinct costs on the edges. I designate an edge e. Give an algorithm with running time $\mathcal{O}(m + n)$ to decide whether e is in the minimum spanning tree of G.

2. Suppose you are reading a collection of stories about mythology and have identified n main characters, all of whom lived a long long time ago. However our favorite archaeology professor[1] warned us, "70% of all archaeology is done in the library. Researching. Reading. We cannot afford to take mythology at face value."

   Having completed the library portion, we are now trying to see if we can take the mythology at face value. While we have no way of knowing if the stories are or are not true, we want to find out if they are at least *consistent* with one another.

   By a reading of the stories, you have written a series of facts (according to the stories, at least). Each fact has one of two forms:

   - Persons i and k had lifetimes that overlapped at least somewhat. For example, if we are reading Greek Mythology, Daedalus and Icarus would be one such pair.
   - Person i passed away (died) before person k was born. For example, also in Greek Mythology, the life of Perseus is generally considered to pre-date the life of Heracles.

   For any given pair of people, there is at most one fact in the stories that relates the two.

   Given a collection of stories about n main characters, given an $\mathcal{O}(n^2)$ time algorithm to determine if the stories are consistent: that is, if there is a (relative) timeline of births and deaths that allows every fact we wrote to be true at the same time.

[1] Or at least, your data structure professor's favorite archaeology professor.

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