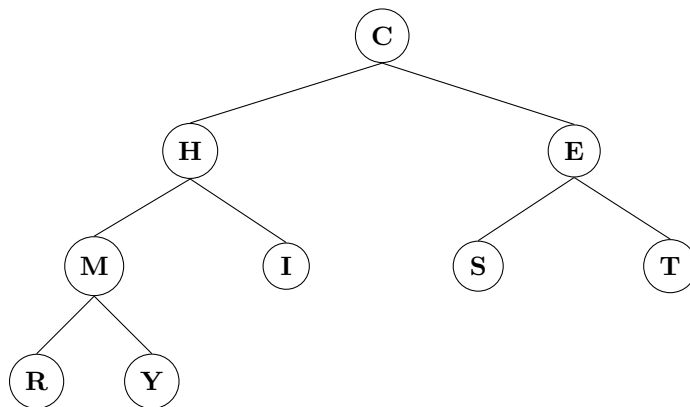A **priority queue** is a data structure that has the following functions available to it:

- `insert(e)` : inserts the given element into the priority queue.

- `min()` : returns a reference to a smallest element in the priority queue.

- `extractMin()` : remove the element that `min()` would return.

We will be discussing implementing the **priority queue interface** with a *binary heap*, which is a *complete binary tree* with the *min-heap property*.

In Lecture, I suggest you focus on understanding how this works conceptually.

For reinforcement after lecture, try to write pseudo-code for these functions based on your understanding. Chapter 8 of textbook of Goodrich and Tamassia has code, as does your Zybook. You will be writing a priority queue as part of project 5.

For demonstration purposes, I am using heaps that store letters of the alphabet, and am using alphabetical order. For those who do not want to sing, alphabetical order is ABCDEFGHIJKLMNOPQRSTUVWXYZ
Question 1. How do we insert a key into a heap? What is the above heap if we insert 'G'?
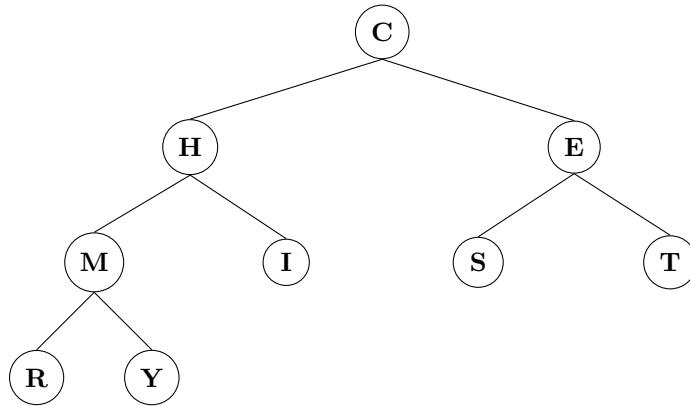*Remember: after inserting, we must still have a heap*

Question 2. From the result of that, insert the keys D, B, and F, in that order. Show the resulting heap and vector after each.

Question 3. How do we implement finding min() in a heap?
Hint: This is the easy part.

Question 4. How do we extract-min from a heap? What do we have after performing an `extract-min` from the following heap?
Remember: after extract-min, we must still have a heap

Height of a Binary Heap

I want to support the claim that a heap with n entries has height $h = \lfloor \log n \rfloor$.
Question 5. Suppose a heap has n entries. For any level $i < h$, how many nodes are in level i? Why?

Question 6. How many *total* entries are there in levels $0 \ldots h - 1$? That is, how many nodes are in all the levels, except the last one?

Question 7. How many entries are in the last level of the heap?

Question 8. Provide an inequality that bounds n, the total number of entries, in terms of h, the height of the heap.

Question 9. What is the running time for the three key functions of the Priority Queue interface, if it is implemented as a binary heap?