In today's lecture, we're going to discuss using graphs to solve problems. This is known as a "reduction" but don't let the fancy name fool you – you are familiar with the basics of the technique by knowing about software libraries.

The first problem used to be a homework problem in this class.

1. We have two containers: one has a capacity of three gallons of water, the other five gallons. Both are initially empty, although we have access to a large water fountain. We can take a non-full container and fill it completely up with the water, or we can completely empty a container (with water in it) into the fountain, or we can pour the contents of one container into the other until either the first is empty or the second is full. Our goal is to find a sequence of actions we can take to end up with exactly four gallons of water in one container (and none in the other container).

   (a) How can we represent this as a graph? You should specify what the vertices and edges represent, how many nodes there would be, and whether the edges are directed or undirected.

   (b) If you drew out the entire graph, how would you find the solution to the problem? Explain specifically what you would be looking for; you do not need to provide code or an algorithm, just a statement of what piece(s) of the graph your code would try to find. For example, if you would look to see if there are at least ten vertices, state that (this is not the correct solution, of course).

2. A number maze is an $n \times n$ grid of positive integers. A token starts in the upper left corner; your goal is to move the token to the lower-right corner. On each turn, you are allowed to move the token up, down, left, or right; the distance you may move the token is determined by the number on its current square. For example, if the token is on a square labeled 3, then you may move the token three steps up, three steps down, three steps left, or three steps right. However, you are never allowed to move the token off the edge of the board. Your goal is to find the minimum number of moves required to solve a given number maze, or to correctly report that the maze has no solution.

   For example, in the following maze, we can solve this with eight moves: right, down, left, right, up, left, right, down.

   Describe how to use a graph to solve this problem. A complete answer includes a description of how to form a graph from an arbitrary number maze as well as how to find a solution to the number maze using the graph, or to report that none is possible.

3	5	7	4	6
5	3	1	5	3
2	8	3	1	4
4	5	7	2	3
3	1	3	2	