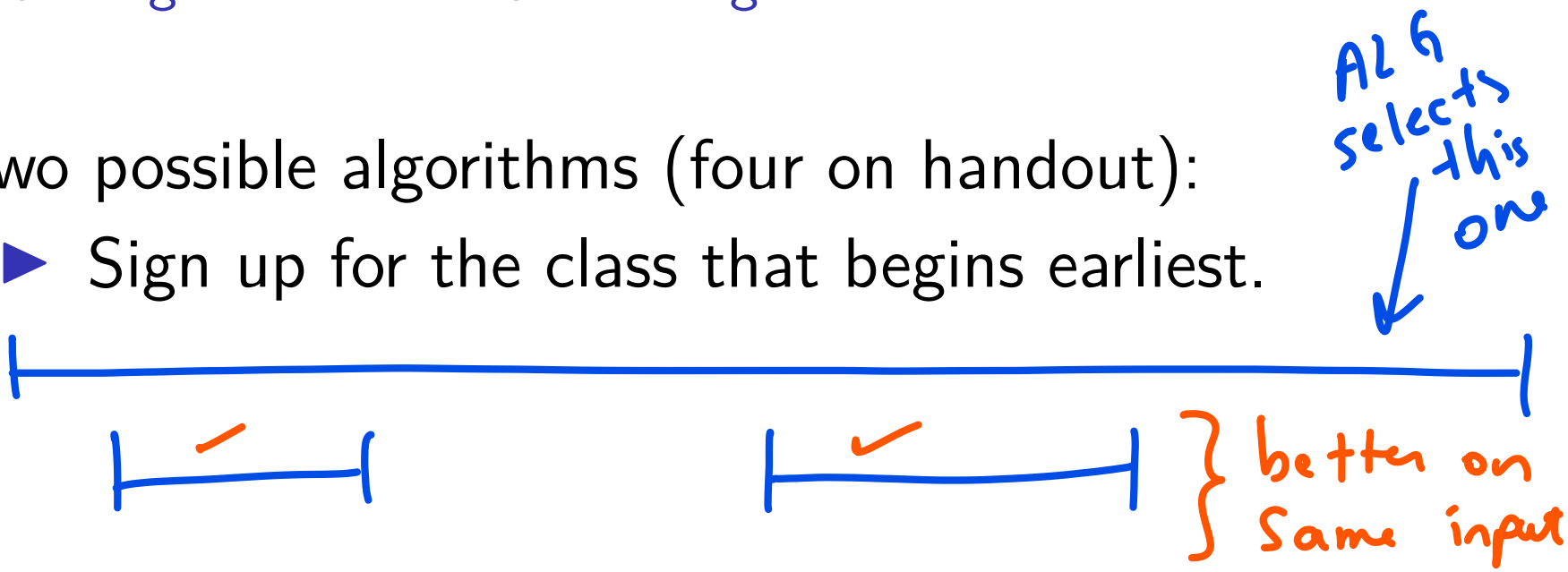
CompSci 260P
Spring 2023 Lecture 10:
Intro to Greedy
Interval Problems and Proofs

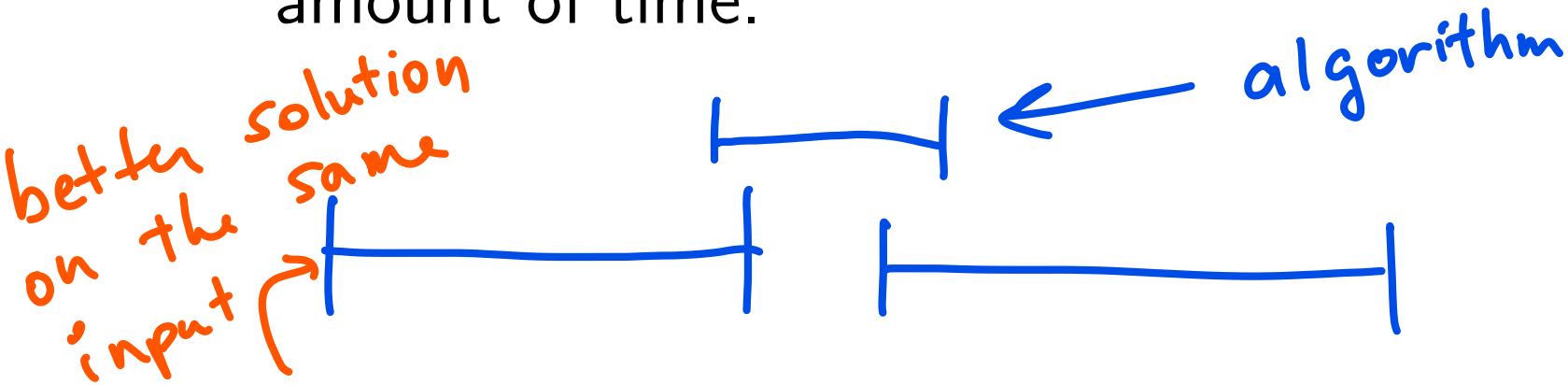
Unweighted Interval Scheduling Problem

Two possible algorithms (four on handout):

- Sign up for the class that begins earliest.



- Sign up for the class that meets for the least amount of time.

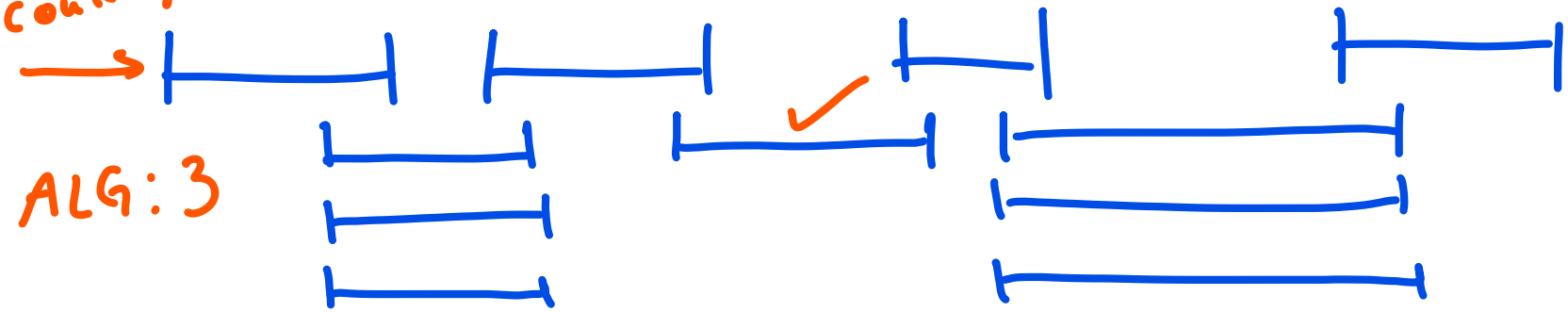


Unweighted Interval Scheduling Problem

Two more algorithms (four on handout):

- Sign up for the class that conflicts with the fewest other classes.

could get 4



ALG: 3

- Sign up for the class that ends earliest.

Interval Scheduling Problem (proof)

Correct Algorithm:

- Sign up for the class that ends earliest.
- Remove it and all overlapping classes from the set of available classes.
- Repeat this process until no classes remain.

Claim: There is an optimal solution that includes the first-ending class.

Proof of Claim: Suppose all optimal solutions do not. Select an arbitrary optimal solution OPT.

output not alg.

"original" optimal.

OPT' = OPT − ? + ??

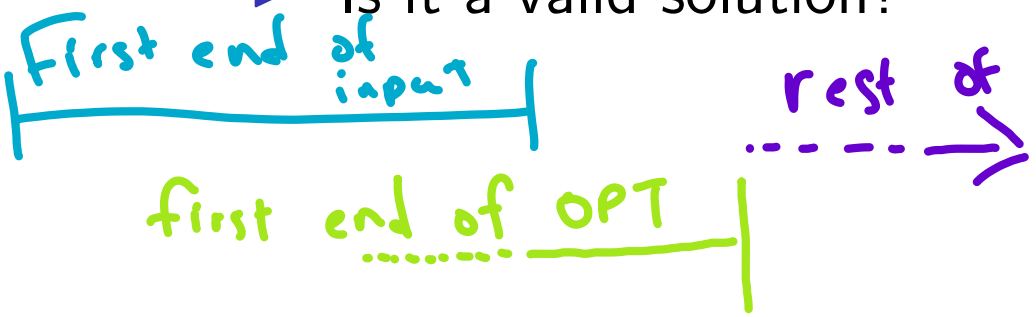
first ending from

Proof of Correctness

- We began with an arbitrary optimal set OPT
  - Its first element was not first-ending. (else, solved already)
  - We removed that one
  - We added our first one: the first-ending.
  - This forms a set we'll call OPT′
- **Claim:** OPT′ is an optimal solution.
  - Is it the same size as every optimal solution?

    Yes; zero net change

  - Is it a valid solution?



Proof of Correctness

- We proved that an optimal solution that includes the first-ending class.
- What does the full proof look like?

Induction (formally).

Comp Sci 260P?

Prove relevant claim.

Interval Coloring

- n groups requested to use a study room
- group i would like to use it from s_i to f_i.
- Cannot put overlapping in same room
- Cannot reject a group.
- Minimize number of distinct rooms assigned
- Why is your algorithm correct?

Interval Coloring Solution

Algorithm: When a group arrives, give lowest numbered room currently free.

- How many rooms will this use?

  maximum overlap

- Could any solution use fewer? Why or why not?

  no; at time of max overlap, you'd have rejected a group or made a shared room.

After Lecture Follow-Up

- Write solutions to today's problems
- Lecture covered the idea
- Written communication $\neq$ spoken presentation
- Share and critique with study group