Before Friday's 6:00 PM lab, I encourage you to try each of these, either alone or with friends in the class. I will present solutions to these in lab, along with tips for how to solve problems like them.

1. The SHARING problem is as follows. You are given non-negative integers $x_1, \ldots, x_n$; you want to decide whether the numbers can be split into two sets S_1 and S_2 with the same sum:

$$\sum_{x_i \in S_1} x_i = \sum_{x_j \in S_2} x_j$$

   You must split them with the property that $S_1 \cup S_2 = x_1, \ldots, x_n$ and that $S_1 \cap S_2 = \emptyset$. That is, each element appears in *exactly one* of the sets.

   Prove that SHARING is $\mathcal{NP}$-complete.

2. [Problem 8-22 in *Algorithm Design*, by Kleinberg and Tardos]

   Suppose that someone gives you a black-box algorithm $\mathcal{A}$ that takes an undirected graph $G = (V, E)$, and a number k, and behaves as follows.

   - If G is not connected, it simply returns "G is not connected."
   - If G is connected and has an independent set of size at least k, it returns "yes."
   - If G is connected and does not have an independent set of size at least k, it returns "no."

   Suppose that the algorithm $\mathcal{A}$ runs in time polynomial in the size of G and k.

   Show how, using calls to $\mathcal{A}$, you could then solve INDEPENDENT SET in polynomial time: Given an arbitrary undirected graph G, and a number k, does G contain an independent set of size at least k?

3. A faculty committee has n members and is going to vote on t issues during the upcoming summer. On every issue, every professor on the committee can vote "Yes" "no" or abstain (this means they do not vote on that issue). Each issue passes if it receives more "yes" votes as it does "no" votes (abstained votes do not count, and if everyone abstains, the issue fails).

   The INNER CIRCLE problem is this. The input is a table that tells us how every professor on the committee voted on each issue, along with an integer value k. We say a subset P' of the professors is an *inner circle* if, for every issue, we can look solely at set P' to determine the result of the vote – each issue passes if and only if it would have passed had only P' voted on it.

   Show that the problem of determining if there is an inner circle of size k is $\mathcal{NP}$-complete.