What have we seen?

- $\mathbb{N} = \{0, 1, 2, \ldots\}$
- $|\mathbb{N}| = \aleph_0$
- There are this many odds, integers ($\mathbb{Z}$)
- There are this many triples of naturals
- There are this many rational numbers

And now $\mathbb{R}$

- $\delta(k, x)$: kth digit of x after decimal point
  - ex: $\delta(2, \pi) = 4$

FSOC

- I claim $f : \mathbb{N} \to \mathbb{R}$ is bijective.

\# I'm thinking of

- To disprove: find $r \in \mathbb{R}$ s.t. $\nexists y f(y) = r$

is $\lfloor f(0) \rfloor = 6$? If so, $\lfloor r \rfloor = 7$

else $\lfloor r \rfloor = 6$

for all $k \in \mathbb{N}, k \geq 1$?

set $\delta(k, r) = 7$ if $\delta(k, y) = 6$

else set $= 6$

Cantor's Theorem

Claim: $|\mathcal{P}(A)| > |A|$
Suppose FSOC it is not. Then there is some $f : A \to 2^A$ that is surjective.

def $B = \{x \in A \mid x \notin f(x)\}$

$B \subseteq A$ so $B \in P(A)$

$B \in 2^A$

$\exists b \quad f(b) = B$?

is $b \in B$?

Yes? $\to\leftarrow$

No? $\to\leftarrow$

CompSci 162
Spring 2023 Lecture 3:
Formal Languages, Automata

Five Languages

$L_1 = \{a, abb, aaaa\}$ finite

all possible strings over an alphabet

$L_2 = \{a^n | n \in \mathbb{N} \text{ is prime}\}$

$L_3 = \{ b^n a^n b^m \mid n, m \in \mathbb{N} \text{ and } n \equiv m \pmod 3) \}$

$L_4 =$ The set of all $w \in \Sigma^*$ with at most three a's

$L_5 = \{a^n \mid n \in \mathbb{N} \text{ and } \exists x, y, z \in \mathbb{N} - \{0\} \text{ such that } x^n + y^n = z^n\} = \{a, aa\}$

Question 9 Which of these languages are infinite?

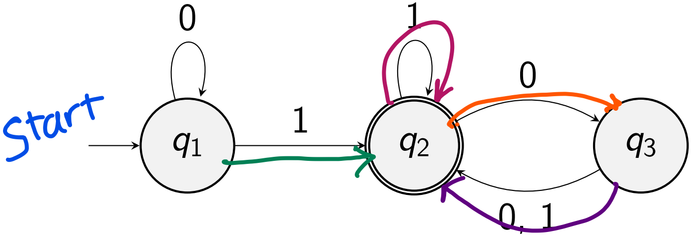
Question 10

Are all languages (over a given alphabet) defined by *some* string in a suitable meta-language?

See Ed discussion

Question 11

What happens when input is "1101"?



◎ = "accept" state

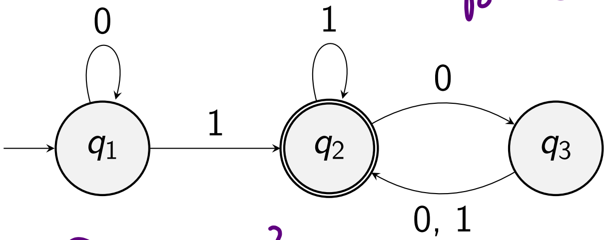
○ = "reject" state (implicit)

This machine accepts 1101

Formal Definition

Q = set of states

q_0 = start state



input alphabet

$Q = \{q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

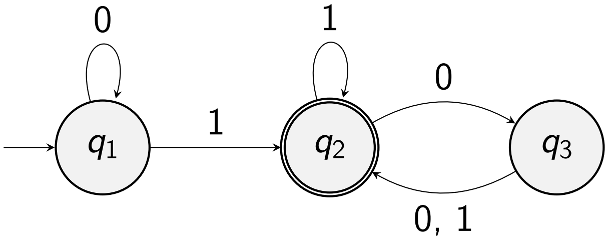
$q_0 = q_1$

$F = \{q_2\}$

$\delta : Q \times \Sigma \mapsto Q$:

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

Formal Definition of Computation



1. $r_0 = q_0$
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for $i = 0 \ldots n-1$
3. $r_n \in F$

accept!

Input = "1101"

1 0

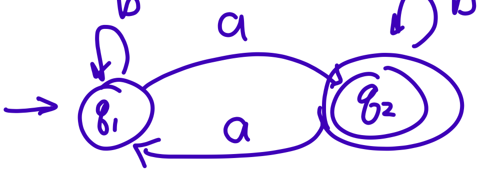
r: q_1 q_2 q_3 q_3 q_2

1 1

Question 13

Design a DFA over the language $\Sigma = \{a, b\}$ that accepts all strings with an odd number of instances of the letter a.

hint: comment your "code"!



// q_1 : even #as

q_2 : odd #as